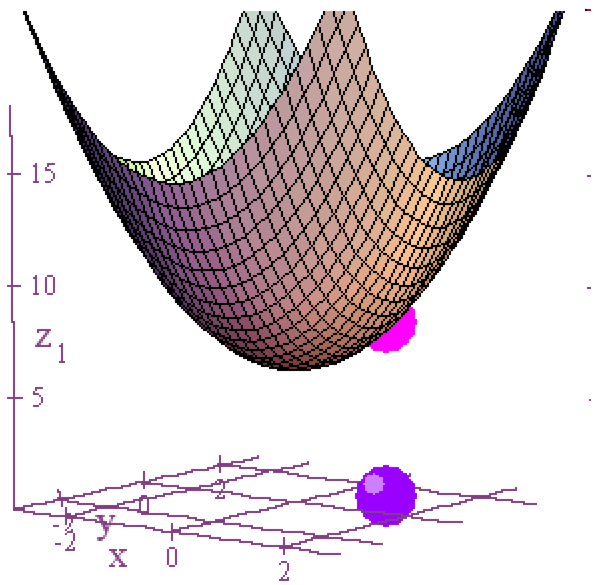
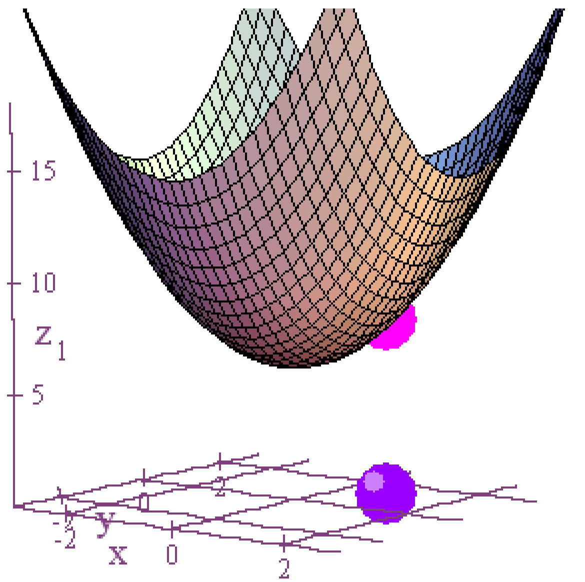
REVIEW



$$z = x^2 + y^2 + 6$$

$$P = (1,1,8)$$

$$Q = (1,1,0)$$



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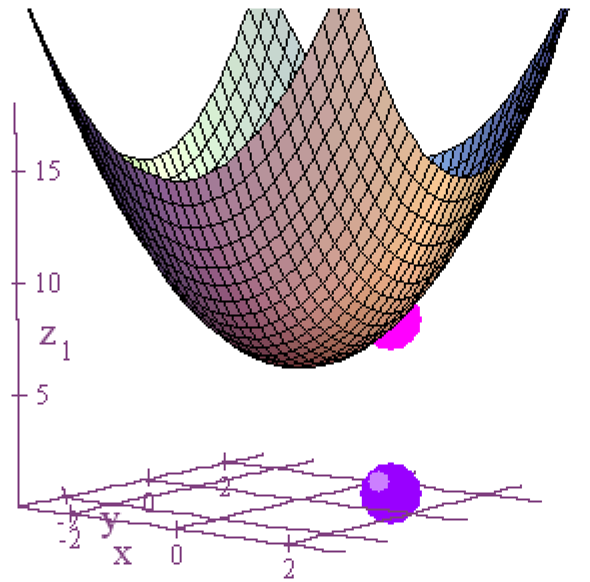
$$\nabla z = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} = 2x\hat{i} + 2y\hat{j}$$

$$\nabla z(1,1) = 2\hat{i} + 2\hat{j}$$



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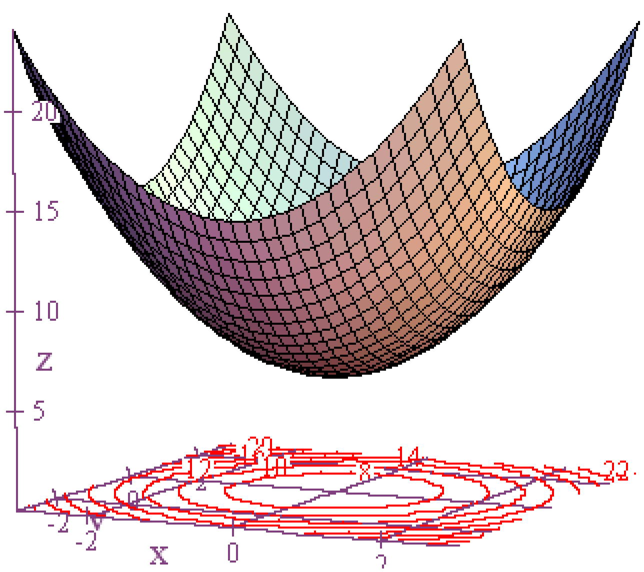

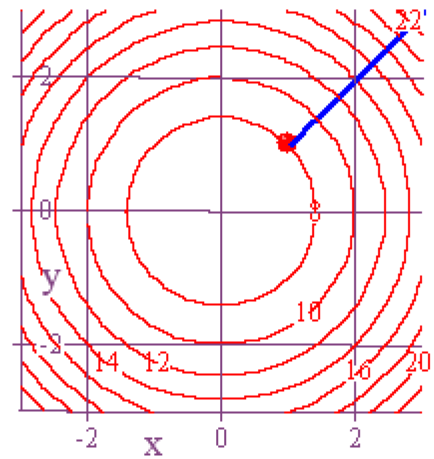

$$x = 1 + 2t$$

$$y = 1 + 2t$$

$$0 \le t \le 1$$

$$0 = x^2 + y^2 - z + 6$$

$$w = x^2 + y^2 - z + 6$$

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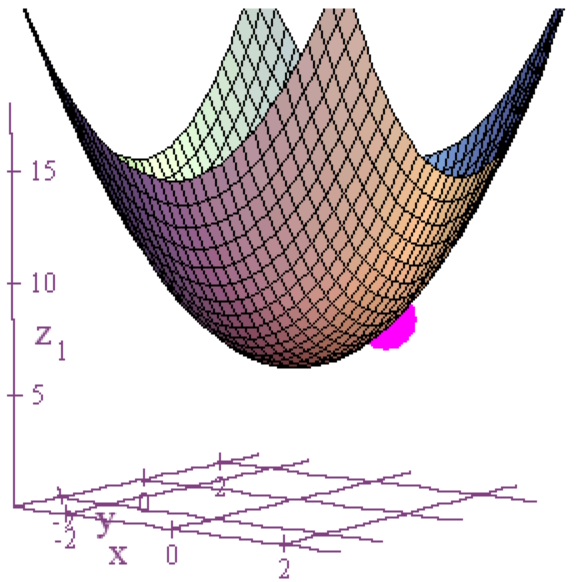

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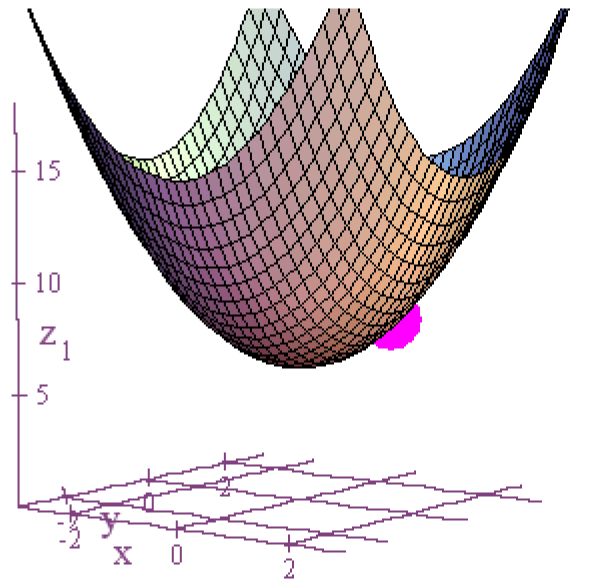
$$\nabla w(1,1,8) = 2\hat{i} + 2\hat{j} - \hat{k}$$



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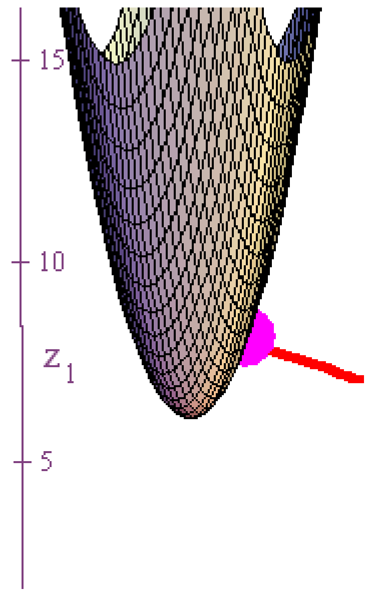

$$x = 1 + 2t$$

$$y = 1 + 2t$$

$$z = 8 - t$$

$$0 \le t \le 1$$

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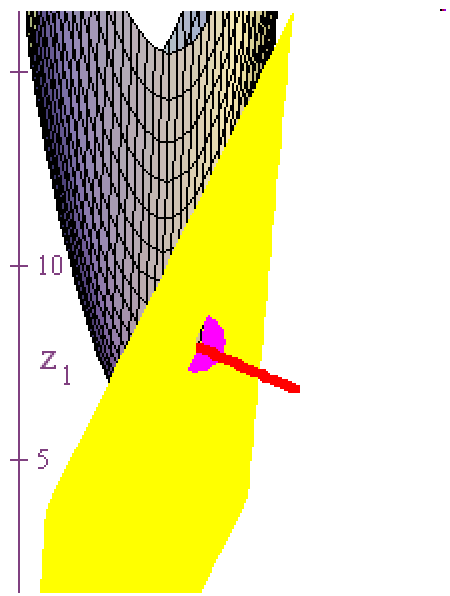

$$x = 1 + 2t$$

$$y = 1 + 2t$$

$$z = 8 - t$$

$$0 \le t \le 1$$

$$z = 2x + 2y + 4$$

$$z = f(x, y) = x^2 + y^2 + 6$$

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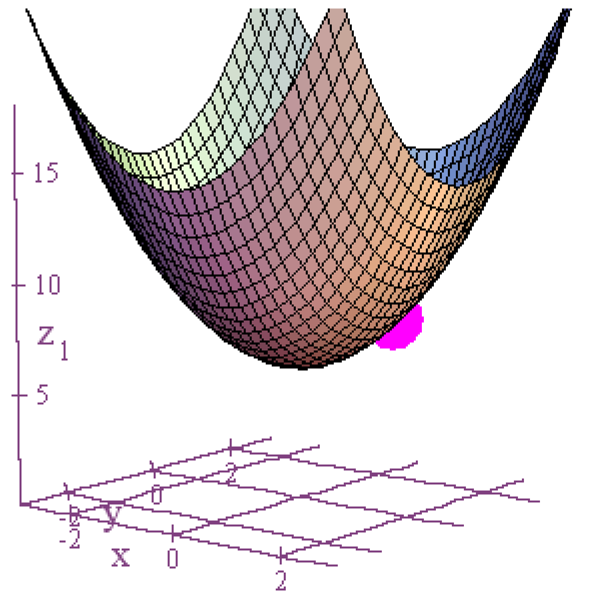


$$\vec{u} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$D_{\vec{u}} f = \nabla f \bullet \vec{u} = \frac{2x}{\sqrt{2}} + \frac{2y}{\sqrt{2}} = \sqrt{2}x + \sqrt{2}y$$

$$D_{\vec{u}} f(1,1) = 2\sqrt{2}$$

$$z = f(x, y) = x^2 + y^2 + 6$$

Construct the tangent line.

$$P = (1,1,8)$$

$$\vec{v} = \hat{i} + \hat{j} + 8\hat{k}$$

$$\nabla z(1,1) = 2\hat{i} + 2\hat{j}$$

$$\vec{u} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$$

$$t\vec{u} = t\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right) = \frac{t}{\sqrt{2}}\hat{i} + \frac{t}{\sqrt{2}}\hat{j}$$

$$D_{\vec{u}} f(1,1) = 2\sqrt{2}$$

$$\vec{w} = 2\sqrt{2}t\hat{k}$$

$$\vec{r}(t) = \vec{v} + t\vec{u} + \vec{w} = \left(1 + \frac{t}{\sqrt{2}}\right)\hat{i} + \left(1 + \frac{t}{\sqrt{2}}\right)\hat{j} + \left(8 + 2\sqrt{2}t\right)\hat{k}$$

$$0 \le t < \infty$$

$$0 = x^2 + y^2 - z + 6$$

$$w = x^2 + y^2 - z + 6$$

$$P = (1,1,8)$$

$$\nabla w = \frac{\partial w}{\partial x}\hat{i} + \frac{\partial w}{\partial y}\hat{j} + \frac{\partial w}{\partial z}\hat{k} = 2x\hat{i} + 2y\hat{j} - \hat{k}$$

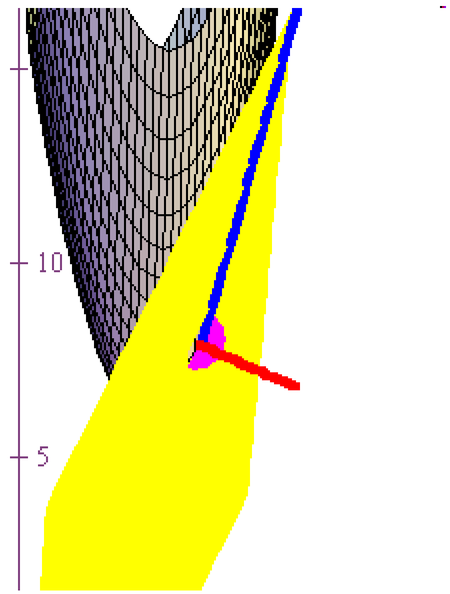
$$\nabla w(1,1,8) = 2\hat{i} + 2\hat{j} - \hat{k}$$

$$x = 1 + \frac{t}{\sqrt{2}}$$

$$y = 1 + \frac{t}{\sqrt{2}}$$

$$z = 8 + 2\sqrt{2}t$$

$$0 \le t < \infty$$



$$x = 1 + 2t$$

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$$0 \le t \le 1$$

$$z = 2x + 2y + 4$$

Recall that:

$$D_{\vec{u}} f = \nabla f \bullet \vec{u} = \left\| \nabla f \right\| \left\| \vec{u} \right\| \cos\theta = \left\| \nabla f \right\| \cos\theta$$

Recall that:

$$D_{\vec{u}} f = \nabla f \bullet \vec{u} = \|\nabla f\| \|\vec{u}\| \cos\theta = \|\nabla f\| \cos\theta$$

Consequently,

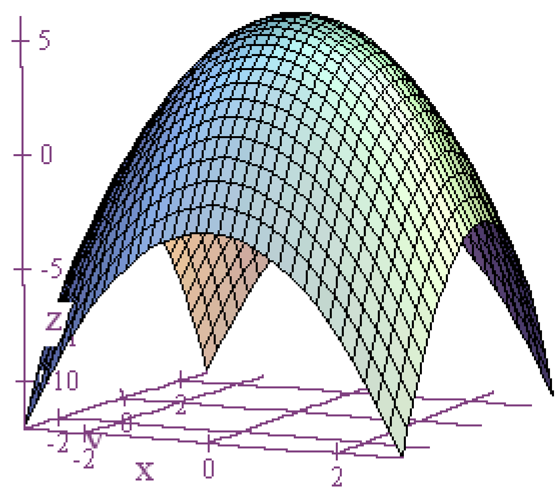
$D_{\vec{u}} f$ is maximized when $\theta = 0$

$D_{\vec{u}} f$ is minimized when $\theta = \pi$

$D_{\vec{u}} f = 0$ when $\nabla f \perp \vec{u}$

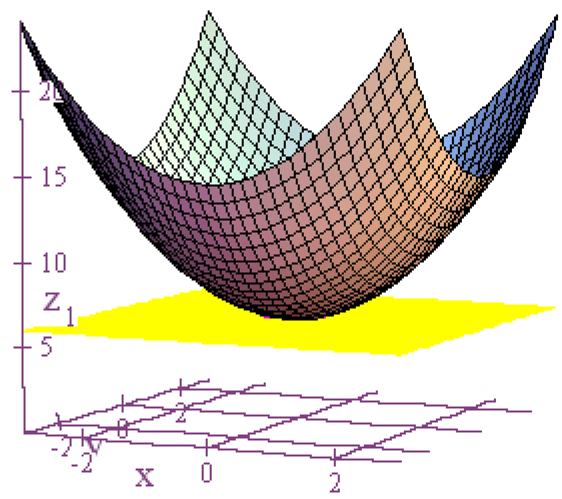
In other words:

1. **You ascend a hill most rapidly when you go in the compass direction indicated by the gradient vector of *z*.**
2. **You descend a hill most rapidly when you go in the compass direction indicated by the negative of the gradient vector of *z*.**
3. **Your elevation doesn't change when you go in a compass direction at right angles to the gradient vector of *z*.**



$$z = x^2 + y^2 + 6$$

Recall that when at a relative maximum or minimum point on a surface, if a tangent plane exists, then it must be horizontal.



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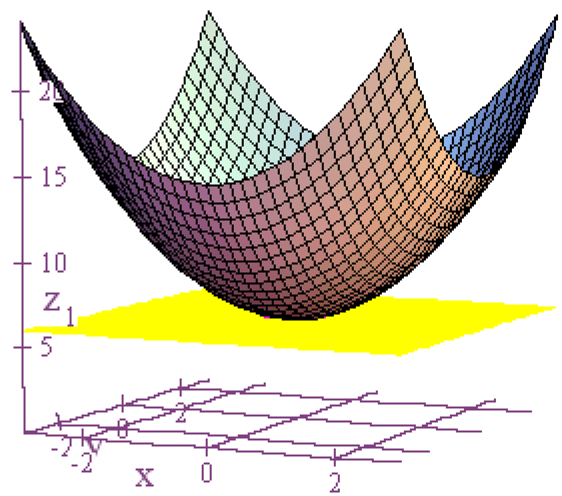
Recall that when at a relative maximum or minimum point on a surface, if a tangent plane exists, then it must be horizontal.

This also suggests that the partial derivatives are zero at such a point.



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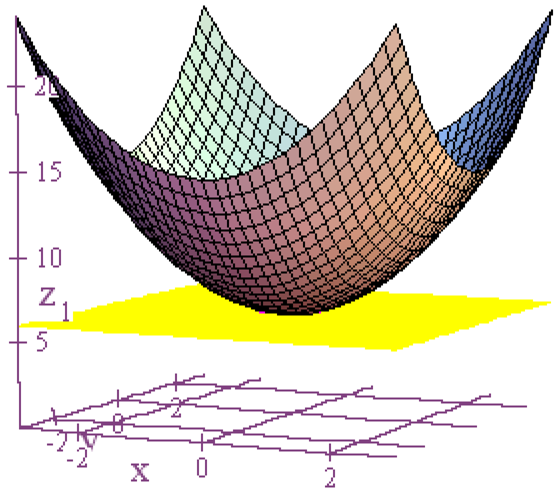
Recall that when at a relative maximum or minimum point on a surface, if a tangent plane exists, then it must be horizontal.

This also suggests that the partial derivatives are zero at such a point.

And this leads to the following test.



Definition: Let (a,b) be a point contained in an open region R on which a functions $z = f(x,y)$ is defined. Then (a,b) is a critical point if any of the following conditions are true:

1. $z_x(a,b) = 0 = z_y(a,b)$
2. $z_x(a,b)$ does not exist
3. $z_y(a,b)$ does not exist

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Theorem: If $z = f(x, y)$ has a local maximum or a local minimum at a point (a,b) contained within an open region R on which $z = f(x, y)$ is defined, then (a,b) is a critical point.

Second Partials Test: Suppose $z = f(x, y)$ has continuous second partial derivatives on an open region containing a point (a,b) such that $z_x(a,b) = 0 = z_y(a,b)$, and let

$$D = D(a,b) = \begin{vmatrix} z_{xx}(a,b) & z_{xy}(a,b) \\ z_{yx}(a,b) & z_{yy}(a,b) \end{vmatrix} = z_{xx}(a,b)z_{yy}(a,b) - z_{xy}(a,b)z_{yx}(a,b).$$

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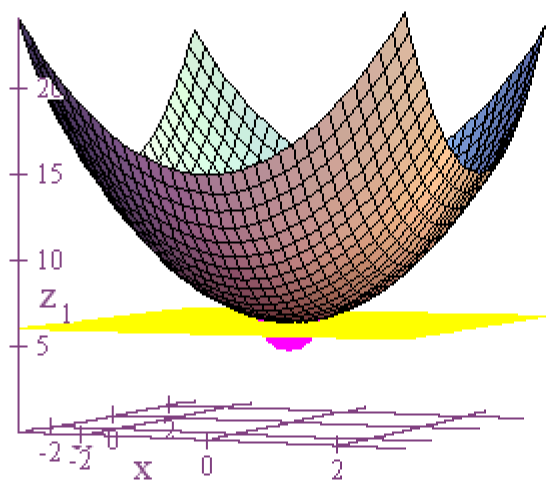
Then:

1. If $D > 0$ and $z_{xx}(a,b) > 0$, $f(a,b)$ is a local minimum.
2. If $D > 0$ and $z_{xx}(a,b) < 0$, $f(a,b)$ is a local maximum.
3. If $D < 0$, $\left(a,b,f(a,b)\right)$ is a saddle point.
4. If $D = 0$, the test is inconclusive.

Hence,

$$z = x^2 + y^2 + 6$$
$$z_x = 2x$$
$$z_y = 2y$$

$$\begin{matrix} z_x = 0 \\ z_y = 0 \end{matrix} \Rightarrow \begin{matrix} 2x = 0 \\ 2y = 0 \end{matrix} \Rightarrow \begin{matrix} x = 0 \\ y = 0 \end{matrix} \Rightarrow \text{Critical Point } = (0,0)$$



$$D = \begin{vmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4$$

$$\begin{matrix} D(0,0) = 4 > 0 \\ z_{xx}(0,0) = 2 > 0 \end{matrix} \Rightarrow (0,0,6) \text{ is a local minimum point}$$