The Total Differential

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Suppose $z=f(x,y)$ is locally linear at (a,b,c).

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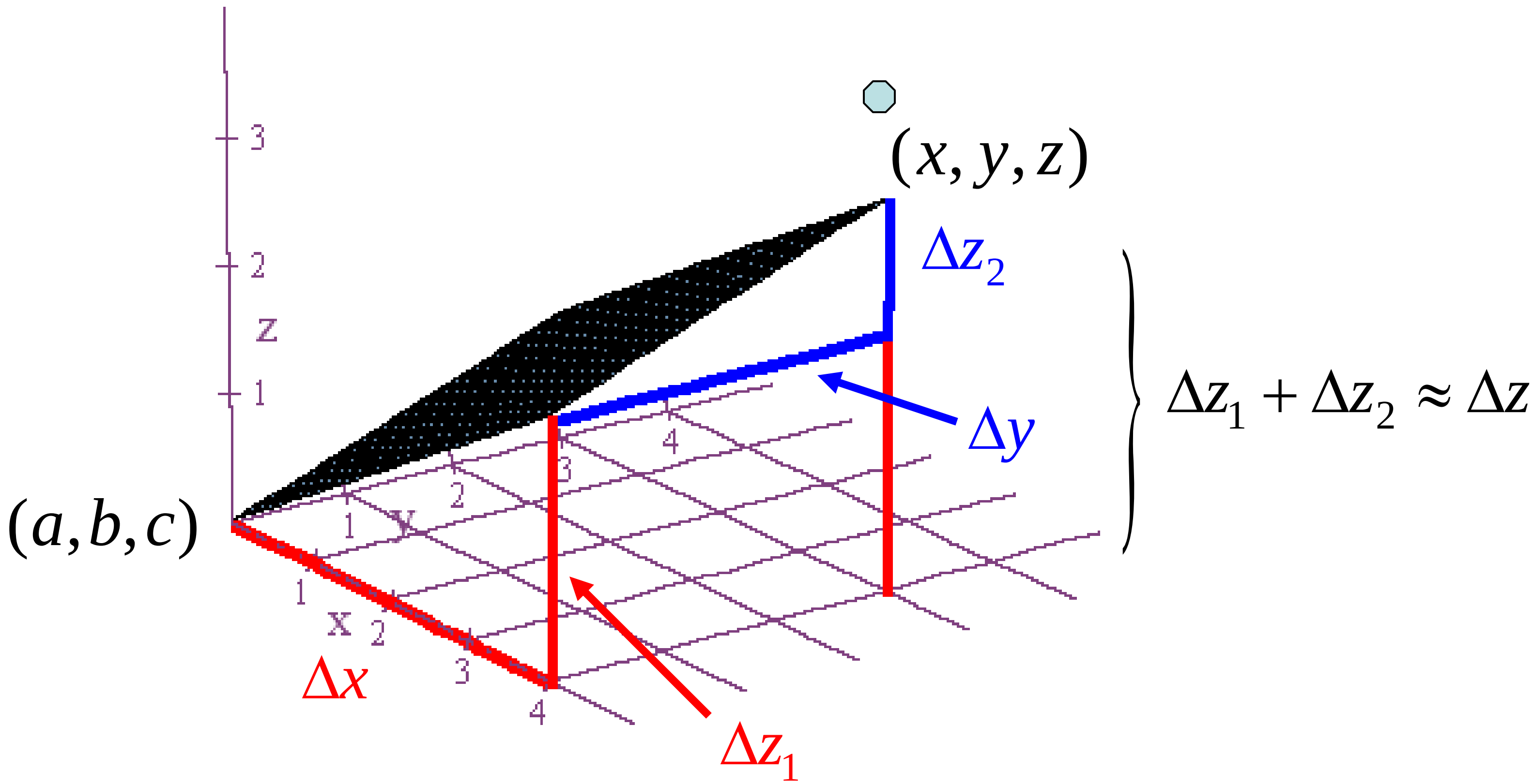
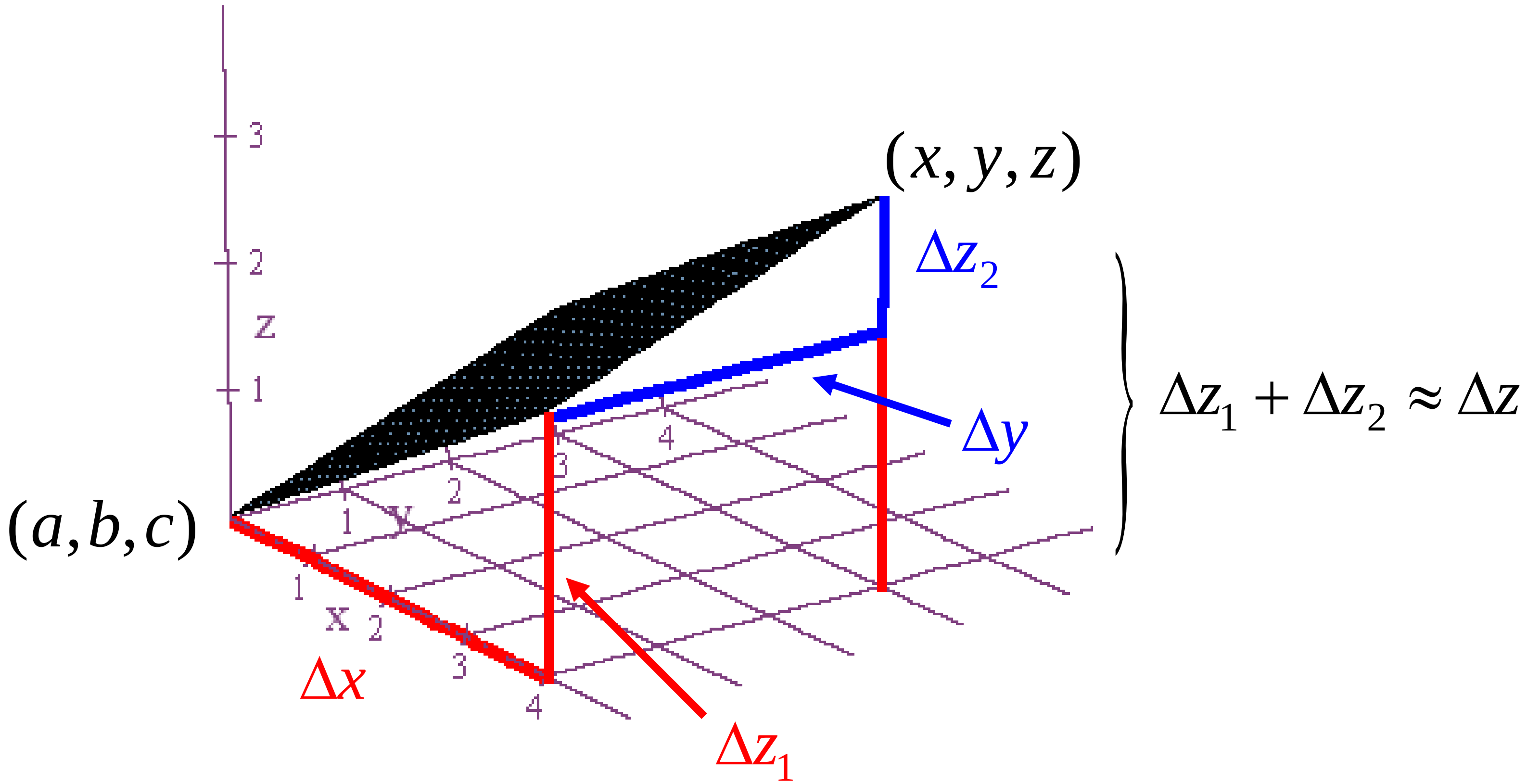
Suppose $z=f(x,y)$ is locally linear at (a,b,c).

Then we can use a plane to approximate the change in z for points near (a,b,c).



(x, y, z)
Δz_2
$\Delta z_1 + \Delta z_2 \approx \Delta z$
Δy
(a, b, c)
Δx
Δz_1
z
y
x

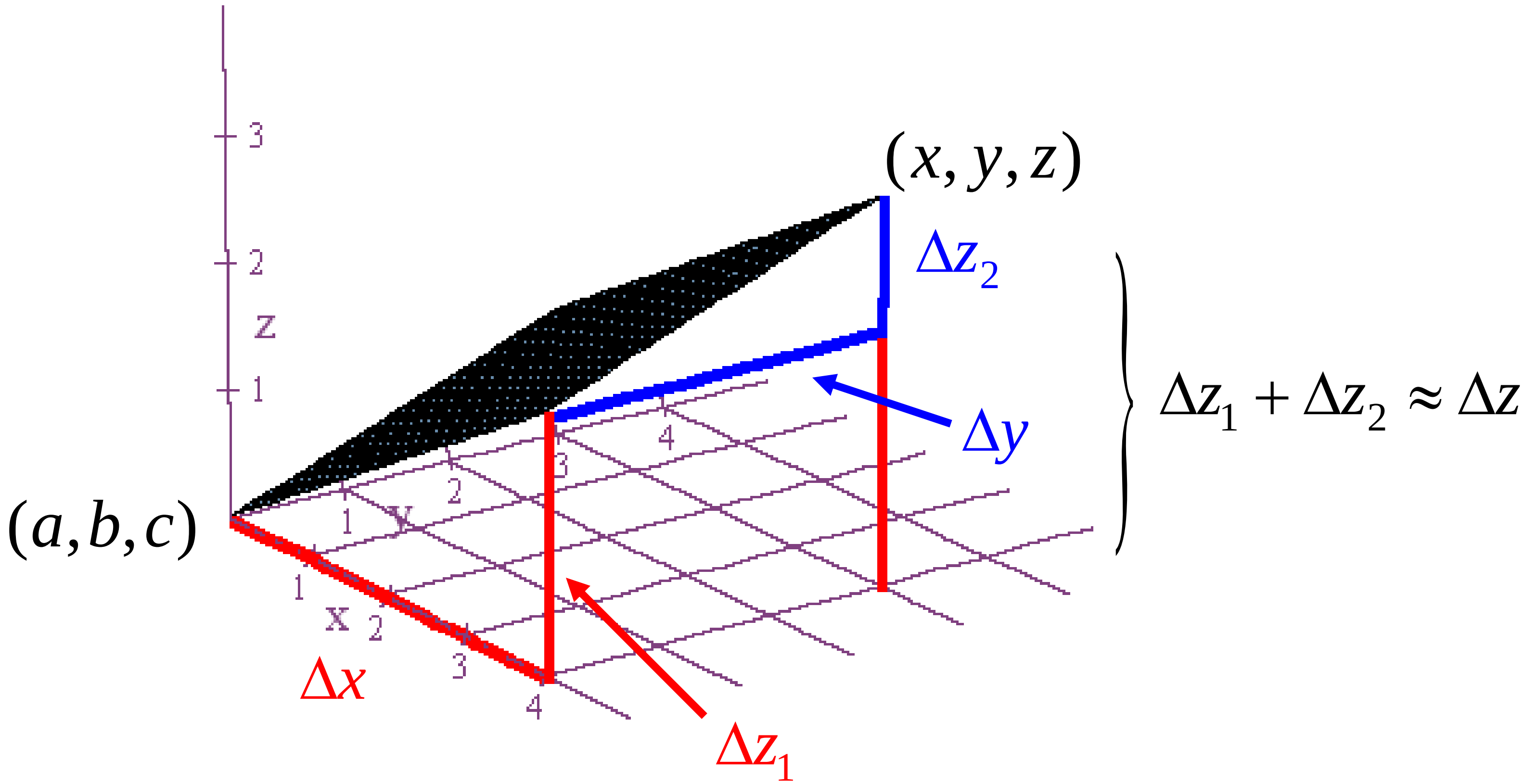
The Total Differential



$$slope_x = m_x = \frac{\Delta z_1}{\Delta x}$$

$$slope_y = m_y = \frac{\Delta z_2}{\Delta y}$$

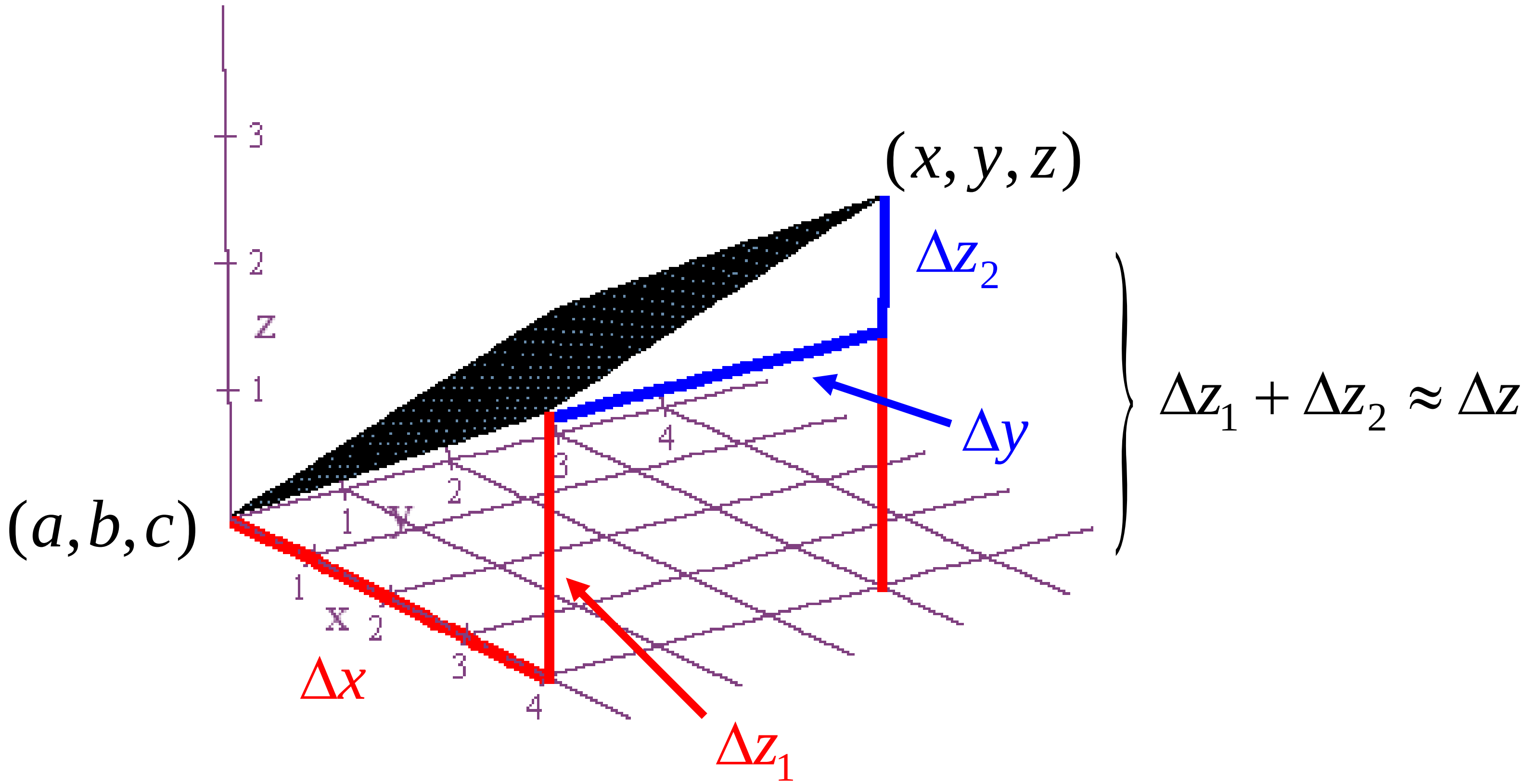
The Total Differential



$$slope_x = m_x = \frac{\Delta z_1}{\Delta x} \Rightarrow \Delta z_1 = m_x \Delta x = \frac{\partial z}{\partial x} \Delta x$$

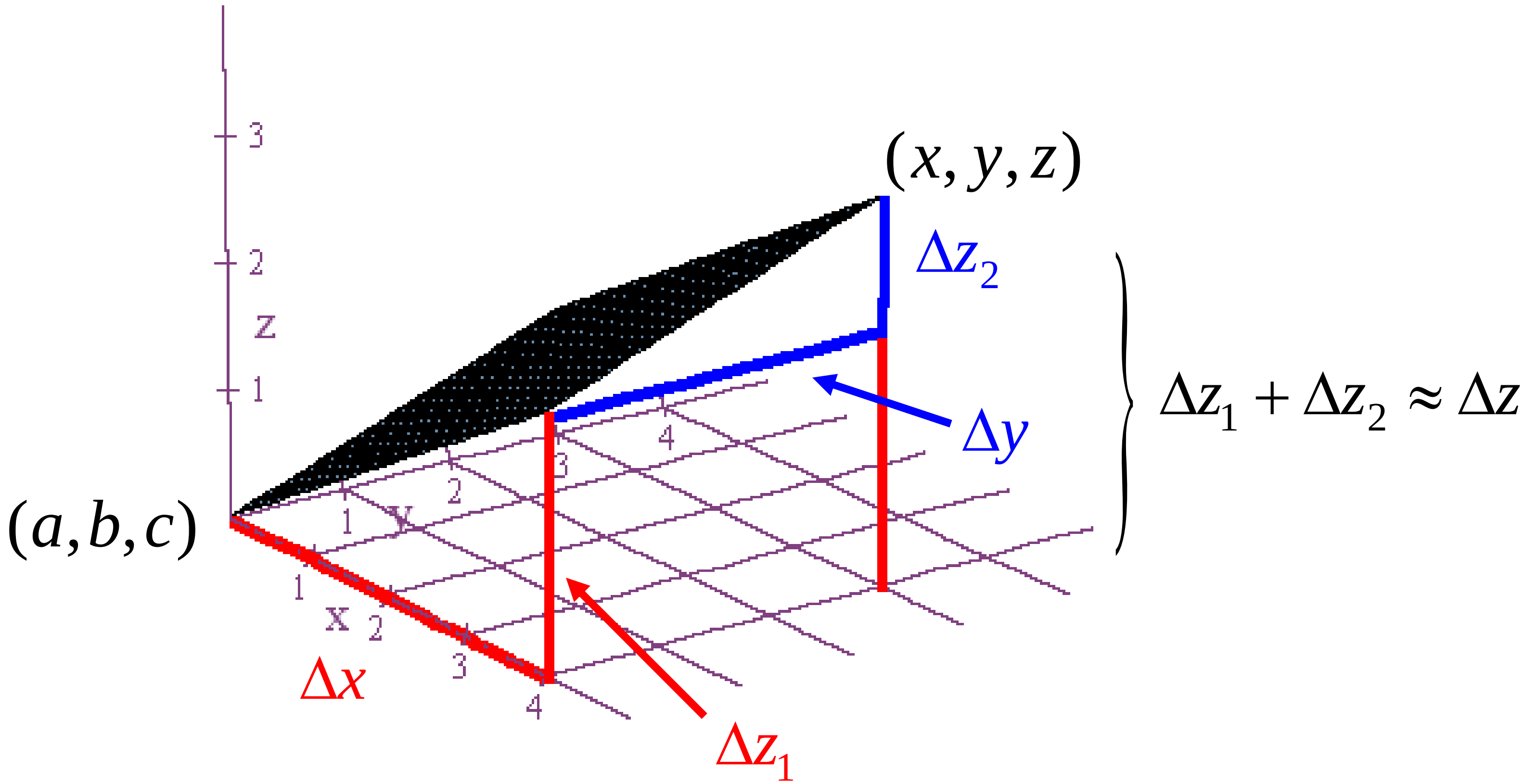
$$slope_y = m_y = \frac{\Delta z_2}{\Delta y} \Rightarrow \Delta z_2 = m_y \Delta y = \frac{\partial z}{\partial y} \Delta y$$

The Total Differential



$$\Delta z \approx \Delta z_1 + \Delta z_2 = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

The Total Differential



$$\Delta z \approx \Delta z_1 + \Delta z_2 = \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

We sometimes use the total differential to tell us how to make substitutions such as you did in single variable calculus.

Single Variable: $u = f(x)$

$$du = f'(x)dx = \frac{df}{dx}dx$$

Multivariable: $z = f(x, y)$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Other times we use the total differential to tell help us approximate either a function value or the change in output.

Single Variable: $u = f(x)$

$$du = f'(x)dx = \frac{df}{dx}dx$$

$$u_2 - u_1 = \Delta u \approx f'(x)\Delta x$$

$$u_2 \approx f'(x)\Delta x + u_1$$

Multivariable: $z = f(x, y)$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$z_2 - z_1 = \Delta z \approx \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y$$

$$z_2 \approx \frac{\partial z}{\partial x}\Delta x + \frac{\partial z}{\partial y}\Delta y + z_1$$