

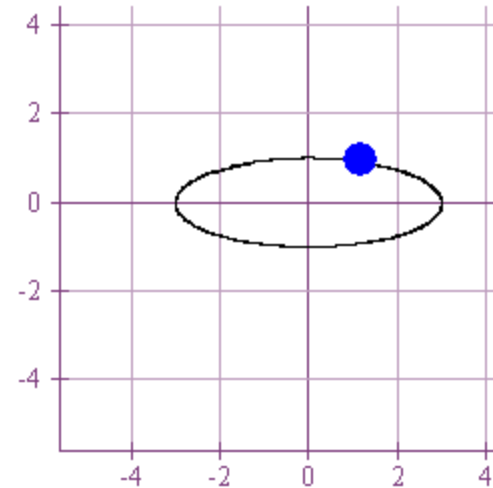
UNIT TANGENTS AND NORMALS



Suppose we have a curve defined by a vector valued function.

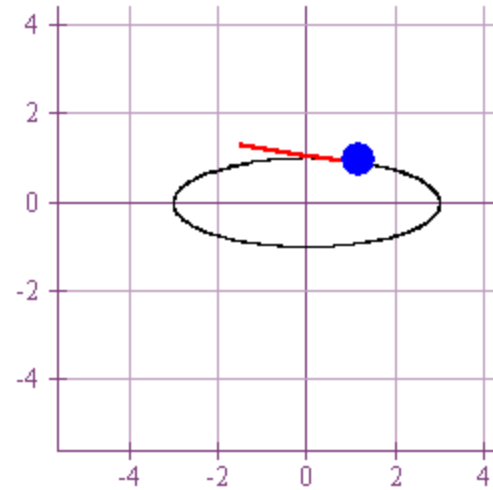
$$\vec{r}(t) = P(t)\hat{i} + Q(t)\hat{j}$$

$$a \leq t \leq b$$



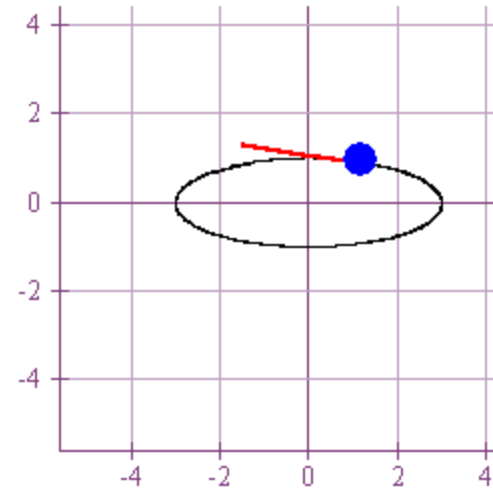
Then the derivative evaluated at a point will give us a vector tangent to the curve.

$$\vec{r}'(t) = P'(t)\hat{i} + Q'(t)\hat{j}$$



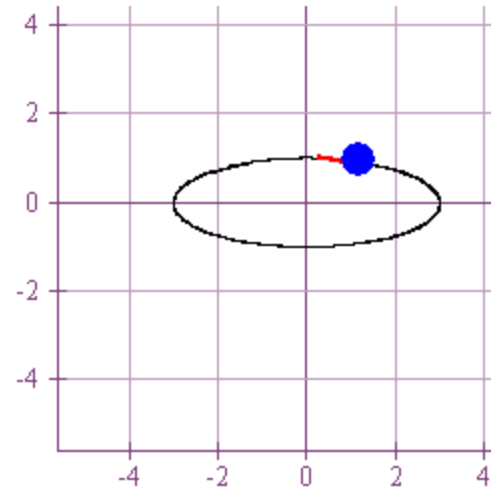
However, this vector is not necessarily a unit vector.

$$\vec{r}'(t) = P'(t)\hat{i} + Q'(t)\hat{j}$$



To get the unit tangent vector, divide $d\vec{r}/dt$ by its length.

$$T = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{d\vec{r}'/dt}{\|d\vec{r}'/dt\|}$$

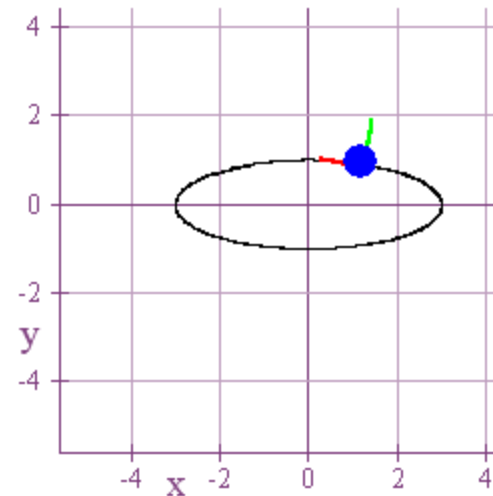


If our unit tangent vector is written in component form as,

$$T = \frac{P'(t)}{\|r'(t)\|} \hat{i} + \frac{Q'(t)}{\|r'(t)\|} \hat{j}$$

Then we define the unit normal by,

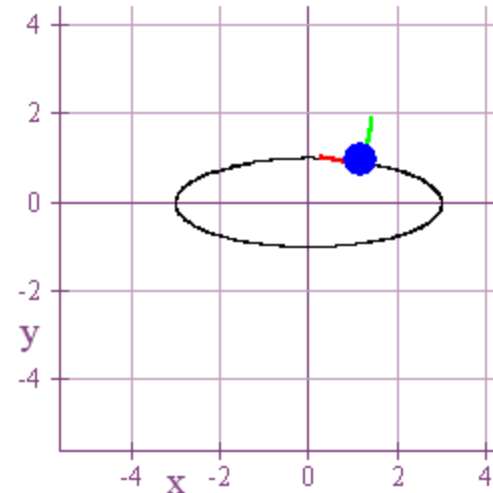
$$N = \frac{Q'(t)}{\|r'(t)\|} \hat{i} - \frac{P'(t)}{\|r'(t)\|} \hat{j}$$



This method always results in a normal vector that points to the right of the direction in which our unit tangent vector is facing.

$$T = \frac{P'(t)}{\|r'(t)\|} \hat{i} + \frac{Q'(t)}{\|r'(t)\|} \hat{j}$$

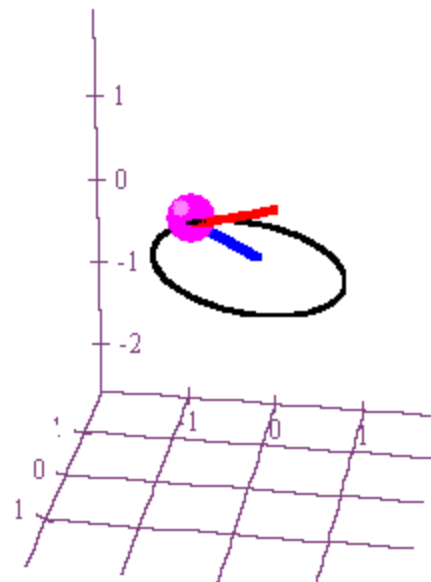
$$N = \frac{Q'(t)}{\|r'(t)\|} \hat{i} - \frac{P'(t)}{\|r'(t)\|} \hat{j}$$



If we have a curve in three dimensions, then things are slightly more complicated.

$$\vec{r}(t) = P(t)\hat{i} + Q(t)\hat{j} + R(t)\hat{k}$$

$$a \leq t \leq b$$



However, since we won't need that case for what we are going to do later, we'll skip it for now.



$$\vec{r}(t) = P(t)\hat{i} + Q(t)\hat{j} + R(t)\hat{k}$$

$$a \leq t \leq b$$

