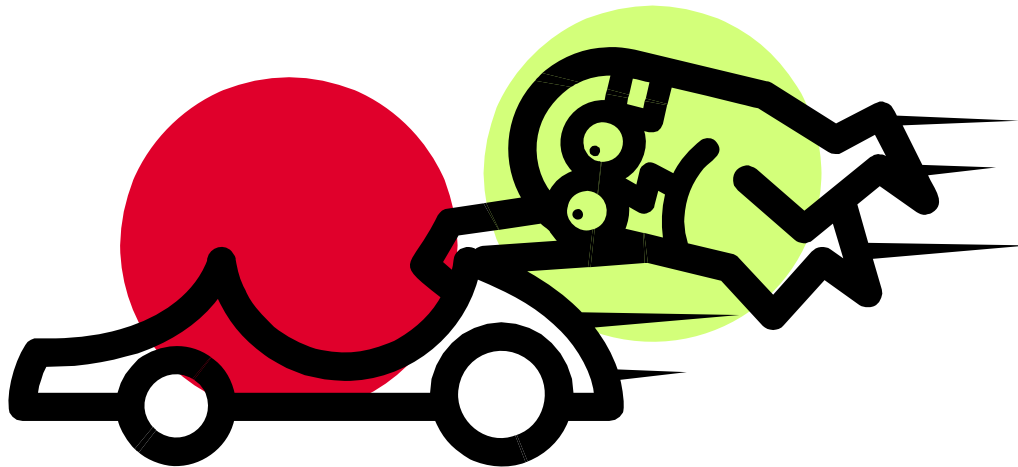


# DERIVATIVES OF VECTOR VALUED FUNCTIONS



**Let**  $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} = \textit{position}$

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**Then,**

$$\vec{r}'(t) = \frac{d\vec{r}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t}$$

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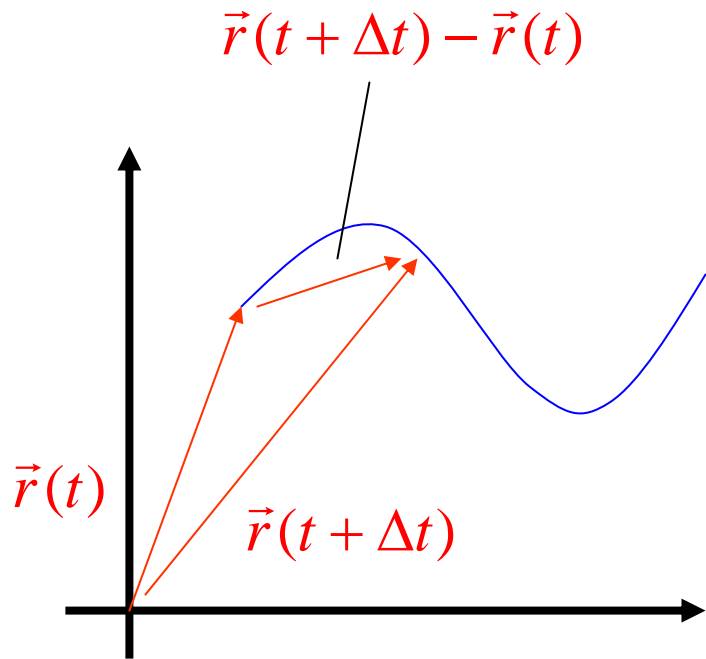
$$\begin{aligned}\vec{r}'(t) &= \frac{d\vec{r}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t} \\ &= \lim_{\Delta t \rightarrow 0} \frac{\left( x(t + \Delta t)\hat{i} + y(t + \Delta t)\hat{j} + z(t + \Delta t)\hat{k} \right) - \left( x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} \right)}{\Delta t}\end{aligned}$$

$$= \lim_{\Delta t \rightarrow 0} \frac{[x(t + \Delta t) - x(t)]\hat{i} + [y(t + \Delta t) - y(t)]\hat{j} + [z(t + \Delta t) - z(t)]\hat{k}}{\Delta t}$$

$$= \lim_{\Delta t \rightarrow 0} \left[ \frac{x(t + \Delta t) - x(t)}{\Delta t} \hat{i} + \frac{y(t + \Delta t) - y(t)}{\Delta t} \hat{j} + \frac{z(t + \Delta t) - z(t)}{\Delta t} \hat{k} \right]$$

$$= \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} = \frac{d\vec{r}}{dt} = \vec{r}'(t) = \vec{v}(t) = \text{velocity}$$

Notice that  $\vec{r}'(t)$  will be tangent to our curve.



$$\vec{r}'(t) = \frac{d\vec{r}}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t}$$

**Also,**

$$\vec{r}''(t) = \frac{d\vec{v}}{dt} = \vec{a}(t) = \textit{acceleration}$$

**And,**

$$\vec{r}'''(t) = \frac{d\vec{a}}{dt} = \textit{the jerk (why?)}$$