INTEGRALS OF VECTOR VALUED FUNCTIONS

Let $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

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Then,

$$\int_a^b \vec{r}(t)\,dt = \lim_{\Delta t \to 0} \sum \vec{r}(t)\,\Delta t$$

$$= \lim_{\Delta t \to 0} \left[\left(\sum x(t)\,\Delta t \right) \hat{i} + \left(\sum y(t)\,\Delta t \right) \hat{j} + \left(\sum z(t)\,\Delta t \right) \hat{k} \right]$$

$$= \left(\int_a^b x(t)\,dt \right) \hat{i} + \left(\int_a^b y(t)\,dt \right) \hat{j} + \left(\int_a^b z(t)\,dt \right) \hat{k}$$

In particular, if *R(t)* is an antiderivative of *r(t)*, then:

$$\int_a^b \vec{r}(t)\,dt = \vec{R}(t)\Big|_a^b = \vec{R}(b) - \vec{R}(a)$$

Also,

$$\int \vec{r}(t)\,dt = \left(\int x(t)\,dt\right)\hat{i} + \left(\int y(t)\,dt\right)\hat{j} + \left(\int z(t)\,dt\right)\hat{k} = \vec{R}(t) + \vec{C}$$