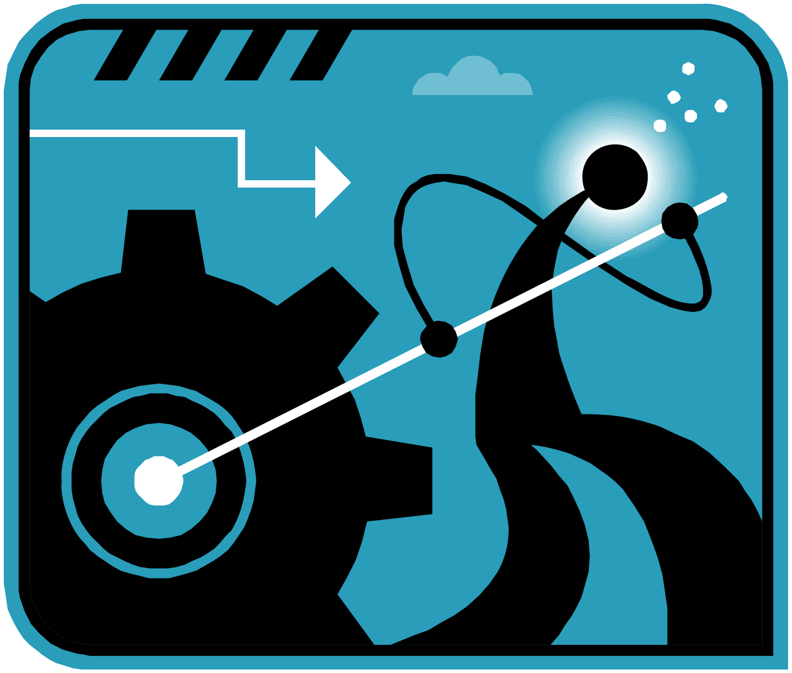
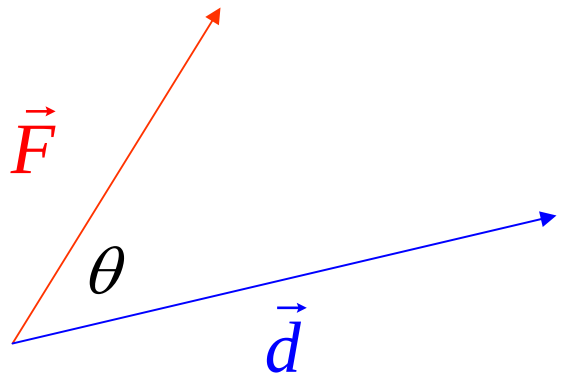
WORK, COMPONENTS, AND PROJECTIONS



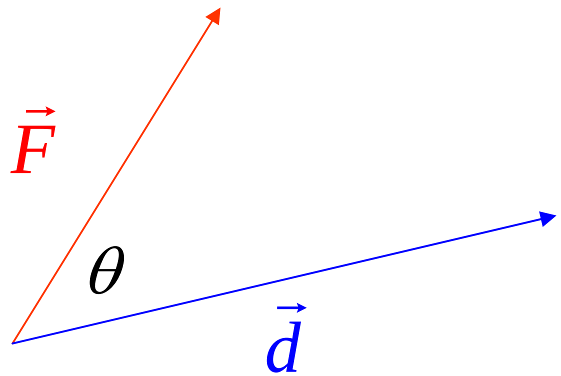
Suppose we have two vectors, $\vec{F} = \hat{i} + 2\hat{j}$ and $\vec{d} = 4\hat{i} + \hat{j}$.



$$\vec{F} = \hat{i} + 2\hat{j}$$

$$\vec{d} = 4\hat{i} + \hat{j}$$

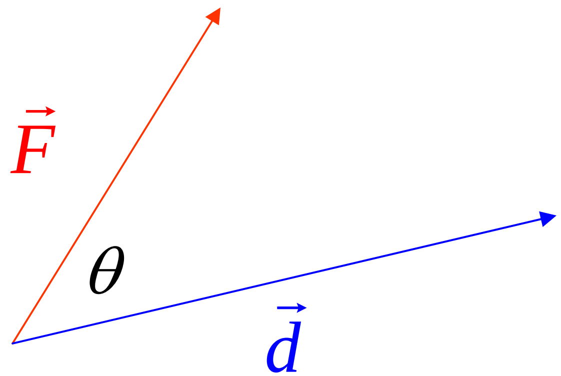
Think of $\vec{F}$ as representing a force in pounds and $\vec{d}$ as representing a distance in feet.



$$\vec{F} = \hat{i} + 2\hat{j}$$

$$\vec{d} = 4\hat{i} + \hat{j}$$

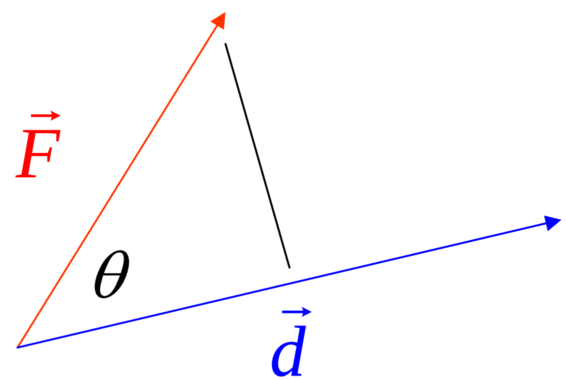
"Work" is classically defined as force×distance.



$$\vec{F} = \hat{i} + 2\hat{j}$$

$$\vec{d} = 4\hat{i} + \hat{j}$$

But what we need to know now is the component of the force $\vec{F}$ that is acting in the direction of the distance vector $\vec{d}$.

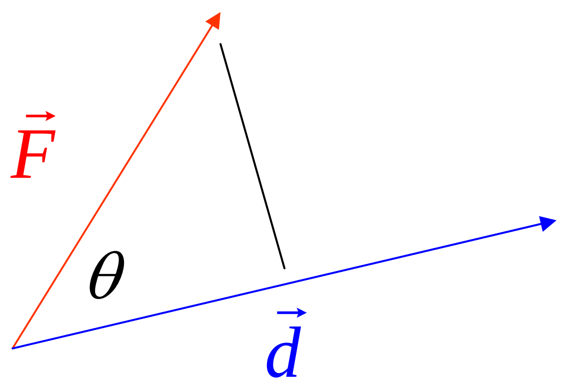


$$\vec{F} = \hat{i} + 2\hat{j}$$

$$\vec{d} = 4\hat{i} + \hat{j}$$

Clearly,

$$comp_{\vec{d}}\vec{F} = \left\|\vec{F}\right\|\cos\theta = \frac{\vec{F}\bullet\vec{d}}{\left\|\vec{d}\right\|} = \frac{6}{\sqrt{17}} = \frac{6\sqrt{17}}{17}$$

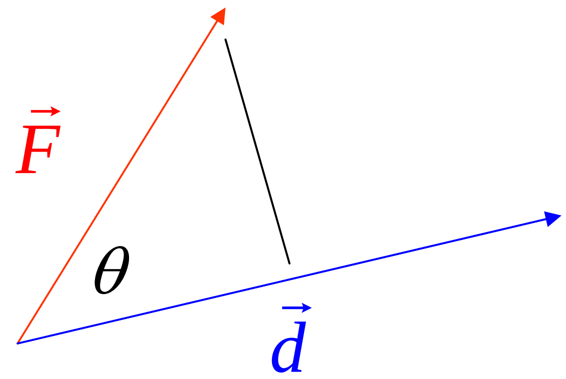


$$\vec{F} = \hat{i} + 2\hat{j}$$

$$\vec{d} = 4\hat{i} + \hat{j}$$

This is a scalar quantity that we call the component of $\vec{F}$ in the direction of $\vec{d}$.

$$comp_{\vec{d}}\vec{F} = \left\|\vec{F}\right\|\cos\theta = \frac{\vec{F}\bullet\vec{d}}{\left\|\vec{d}\right\|} = \frac{6}{\sqrt{17}} = \frac{6\sqrt{17}}{17}$$



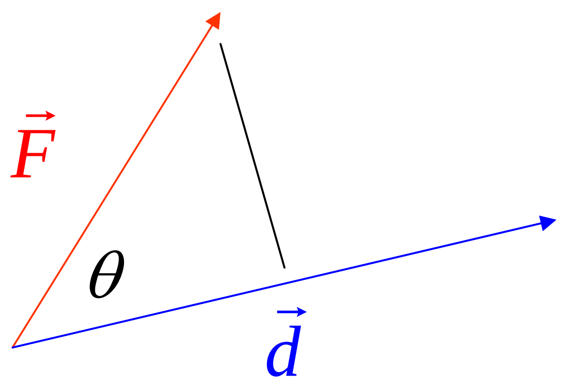
$\vec{F} = \hat{i} + 2\hat{j}$

$\vec{d} = 4\hat{i} + \hat{j}$

To get the corresponding vector, multiply this component by a unit vector in the direction of $\vec{d}$.

$$proj_{\vec{d}}\vec{F} = \left(\left\|\vec{F}\right\|\cos\theta\right)\frac{\vec{d}}{\left\|\vec{d}\right\|} = \frac{\vec{F}\bullet\vec{d}}{\left\|\vec{d}\right\|}\cdot\frac{\vec{d}}{\left\|\vec{d}\right\|} = \left(\frac{\vec{F}\bullet\vec{d}}{\left\|\vec{d}\right\|^2}\right)\vec{d} = \left(\frac{\vec{F}\bullet\vec{d}}{\vec{d}\bullet\vec{d}}\right)\vec{d}$$

$$= \frac{24}{17}\hat{i} + \frac{6}{17}j$$



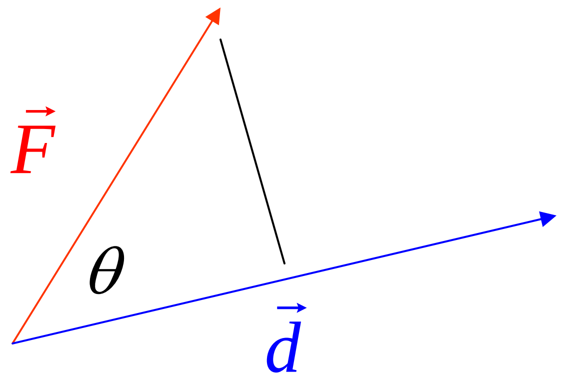
$$\vec{F} = \hat{i} + 2\hat{j}$$

$$\vec{d} = 4\hat{i} + \hat{j}$$

We call this the projection of $\vec{F}$ onto the vector $\vec{d}$.

$$proj_{\vec{d}}\vec{F} = \left(\left\|\vec{F}\right\|\cos\theta\right)\frac{\vec{d}}{\left\|\vec{d}\right\|} = \frac{\vec{F}\bullet\vec{d}}{\left\|\vec{d}\right\|}\cdot\frac{\vec{d}}{\left\|\vec{d}\right\|} = \left(\frac{\vec{F}\bullet\vec{d}}{\left\|\vec{d}\right\|^2}\right)\vec{d} = \left(\frac{\vec{F}\bullet\vec{d}}{\vec{d}\bullet\vec{d}}\right)\vec{d}$$

$$= \frac{24}{17}\hat{i} + \frac{6}{17}j$$

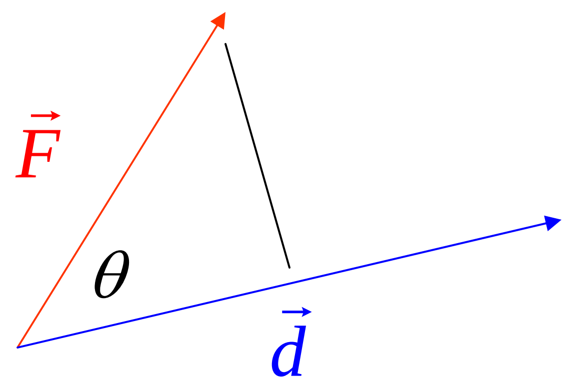


$$\vec{F} = \hat{i} + 2\hat{j}$$

$$\vec{d} = 4\hat{i} + \hat{j}$$

Now to find the work done, take the component of $\vec{F}$ in the dirction of $\vec{d}$ and multiply by the length of $\vec{d}$.

$$work = \left(\left\|\vec{F}\right\|\cos\theta\right)\left\|\vec{d}\right\| = \left\|\vec{F}\right\|\left\|\vec{d}\right\|\cos\theta = \vec{F}\bullet\vec{d} = 6 \text{ foot-pounds}$$



$\vec{F} = \hat{i} + 2\hat{j}$

$\vec{d} = 4\hat{i} + \hat{j}$

And finally, notice that if $\vec{v}$ is vector and $\vec{u}$ is a unit vector,

then $comp_{\vec{u}}\vec{v} == \dfrac{\vec{v}\bullet\vec{u}}{\|\vec{u}\|} = \vec{v}\bullet\vec{u}$.

1. If $\vec{v} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ & $\vec{u} = \hat{i}$, then $comp_{\hat{i}}\vec{v} = \vec{v}\bullet\hat{i} = 2$.

2. If $\vec{v} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ & $\vec{u} = \hat{j}$, then $comp_{\hat{j}}\vec{v} = \vec{v}\bullet\hat{j} = 3$.

3. If $\vec{v} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ & $\vec{u} = \hat{k}$, then $comp_{\hat{k}}\vec{v} = \vec{v}\bullet\hat{k} = 4$.

4. If $\vec{v} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ & $\vec{u} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{k}$,

   then $comp_{\vec{u}}\vec{v} = \vec{v}\bullet\vec{u} = 3\sqrt{2}$.