PRELIMINARIES - ANSWERS

Find the distance between the following points. Give both an exact answer in simplest form and a decimal approximation rounded to the nearest hundredth.

1. $(1,5)$ & $(4,10)$

   distance $=\sqrt{(4-1)^2+(10-5)^2}=\sqrt{3^2+5^2}=\sqrt{34}\approx 5.83$

2. $(-2,3)$ & $(-5,-8)$

   distance $=\sqrt{(-5+2)^2+(-8-3)^2}=\sqrt{(-3)^2+(-11)^2}=\sqrt{130}\approx 11.40$

Find the equation in standard form for the circle with the given center and radius.

3. $(1,5)$ & $r=3$

   $(x-1)^2+(y-5)^2=9$

4. $(-2,3)$ & $r=2$

   $(x+2)^2+(y-3)^2=4$

5. $(0,0)$ & $r=1$

   $x^2+y^2=1$

Complete the square to write the equation for the circle in standard form. Identify the center and radius. Use exact numbers.

6. $x^2+y^2+4x+10y+12=0$

   $(x^2+4x+2^2)+(y^2+10y+5^2)=-12+4+25$

   $(x+2)^2+(y+5)^2=17$

   center $=(-2,-5)$

   radius = $\sqrt{17}$

7. $4x^2+4y^2-16x+32y-24=0$

   $x^2+y^2-4x+8y-6=0$

   $(x^2-4x+(-2)^2)+(y^2+8y+4^2)=6+4+16$

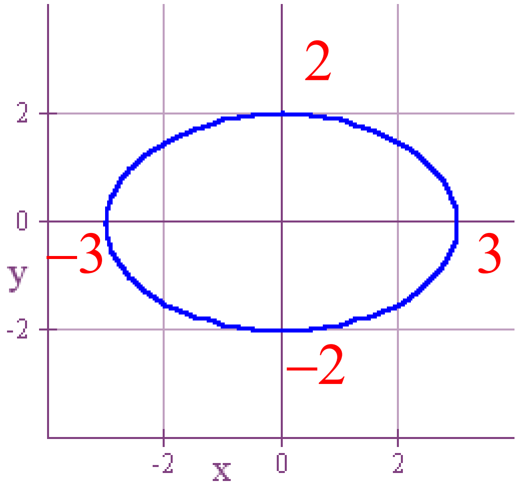
   $(x-2)^2+(y+4)^2=26$

   center $=(2,-4)$

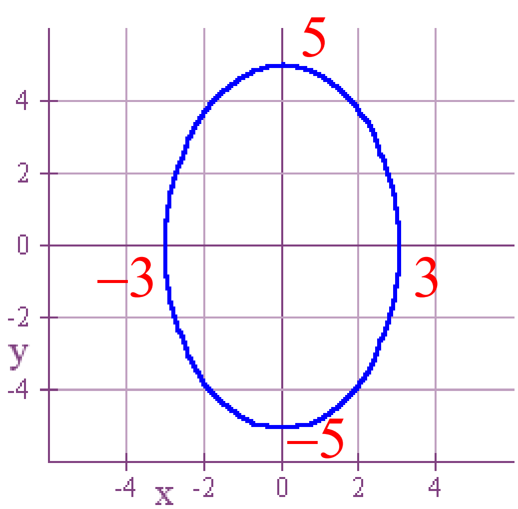
   radius = $\sqrt{26}$

Graph each ellipse, and give the *x* & *y*-intercepts.

8. $\frac{x^2}{9}+\frac{y^2}{4}=1$

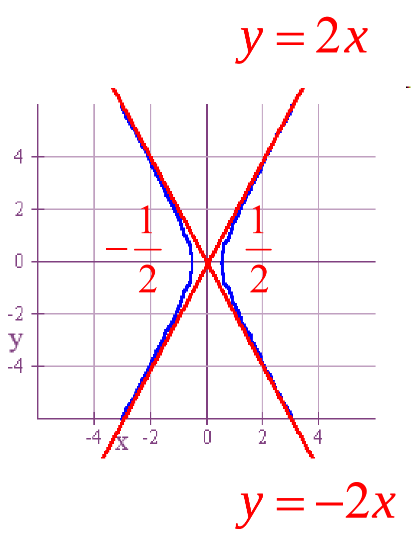


9. $25x^2+9y^2=225 \Rightarrow \frac{25x^2}{225}+\frac{9y^2}{225}=1 \Rightarrow \frac{x^2}{9}+\frac{y^2}{25}=1$

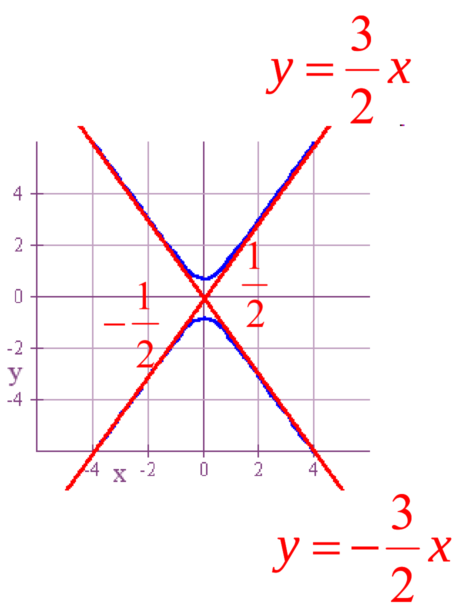


Graph each hyperbola and the corresponding asymptotes. Give the equations of the asymptotes and label all intercepts.

10. $4x^2 - y^2 = 1 \Rightarrow y^2 = 4x^2 - 1 \Rightarrow y = \pm\sqrt{4x^2 - 1} \Rightarrow y = \pm 2x\sqrt{1 - \frac{1}{4x^2}}$



11. $4y^2 - 9x^2 = 1 \Rightarrow y^2 = \frac{1}{4} + \frac{9x^2}{4} \Rightarrow y = \pm\sqrt{\frac{1}{4} + \frac{9x^2}{4}} \Rightarrow y = \pm\frac{3}{2}x\sqrt{\frac{1}{9x^2} + 1}$



Complete the following tables using exact values.

12.

degrees	0	30	45	60	90
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cotangent	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
secant	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
cosecant	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

13.

radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
sine	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
tangent	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cotangent	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
secant	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
cosecant	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Use the identities $\cos^2\theta + \sin^2\theta = 1$ **&** $\cos(a+b) = \cos a\cos b - \sin a\sin b$**, as needed, to verify the following identities.**

14. $1+\tan^2\theta = \sec^2\theta$

$$1+\tan^2\theta = 1+\frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$$

15. $\cot^2\theta + 1 = \csc^2\theta$

$$\cot^2\theta + 1 = \frac{\cos^2\theta}{\sin^2\theta}+1 = \frac{\cos^2\theta}{\sin^2\theta}+\frac{\sin^2\theta}{\sin^2\theta} = \frac{\cos^2\theta+\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} = \csc^2\theta$$

16. $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

$$\frac{1+\cos 2\theta}{2} = \frac{1+\cos\theta\cos\theta - \sin\theta\sin\theta}{2} = \frac{1-\sin^2\theta+\cos^2\theta}{2} = \frac{2\cos^2\theta}{2} = \cos^2\theta$$

17. $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$\frac{1-\cos 2\theta}{2} = \frac{1-\left[\cos\theta\cos\theta - \sin\theta\sin\theta\right]}{2} = \frac{1-\cos^2\theta+\sin^2\theta}{2} = \frac{2\sin^2\theta}{2} = \sin^2\theta$$

Give formulas for the following.

18. Area and circumference of a circle

$A = \pi r^2$ & $C = \pi d = 2\pi r$

19. Area of a triangle

$$A = \frac{bh}{2}$$

20. Area of a parallelogram

$A = bh$

21. Area of a trapezoid

$$A = \frac{(b_1+b_2)h}{2}$$

22. Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

Find equations in slope-intercept form (if possible) for the following.

23. The line of slope 3 that passes through the point (1,5).

$y = 3(x-1)+5$

$y = 3x+2$

24. The line that passes through (-2,8) and (4,-5).

$slope = \frac{-5-8}{4-(-2)} = -\frac{13}{6}$

$y = -\frac{13}{6}(x+2)+8 = -\frac{13}{6}x - \frac{13}{3} + \frac{24}{3} = -\frac{13}{6}x + \frac{11}{3}$

$y = -\frac{13}{6}x + \frac{11}{3}$

25. The line that passes through (-2,-10) and (-2,-5).

$slope = \frac{-10+5}{-2+2} = \frac{-5}{0} = undefined$

$x = -2$

26. The line that passes through (-2,-10) and (2,-10).

$slope = \frac{-10+10}{-2-2} = \frac{0}{-4} = 0$

$y = -10$

27. The line that passes through (-2,-10) and is perpendicular to $3x+2y=10$.

$(-2,-10)$

$3x+2y=10 \Rightarrow 2y=-3x+10 \Rightarrow y=-\frac{3}{2}x+5 \Rightarrow slope=\frac{2}{3}$

$y = \frac{2}{3}(x+2)-10 = \frac{2}{3}x + \frac{4}{3} - \frac{30}{3} = \frac{2}{3}x - \frac{26}{3}$

$y = \frac{2}{3}x - \frac{26}{3}$

Find the following.

28. $\frac{d}{dx}\cos x = -\sin x$

29. $\frac{d}{dx}\sin x = \cos x$

30. $\frac{d}{dx}\cos^2 x = 2\cos x(-\sin x) = -2\cos x \sin x$

31. $\frac{d}{dx}\sin^2 x = 2\sin x \cos x$

32. $\frac{d}{dx}\sec x = \sec x \tan x$

33. $\frac{d}{dx}\csc x = -\csc x \cot x$

34. $\frac{d}{dx}\tan x = \sec^2 x$

35. $\frac{d}{dx}\cot x = -\csc^2 x$

36. $\int \cos x\,dx = \sin x + C$

37. $\int \sin x\,dx = -\cos x + C$

38. $\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$, or using parts, $\frac{\sin x \cos x}{2} + \frac{x}{2} + C$

39. $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$, or using parts, $-\frac{\sin x \cos x}{2} + \frac{x}{2} + C$

40. $\int \sec x\,dx = \ln\left|\sec x + \tan x\right| + C$

41. $\int \frac{1}{x^2-1}dx = \int \left(\frac{-1/2}{x+1} + \frac{1/2}{x-1}\right)dx = -\frac{1}{2}\ln\left|x+1\right| + \frac{1}{2}\ln\left|x-1\right| + C = \ln\sqrt{\left|\frac{x-1}{x+1}\right|} + c$

42. $\int \tan x\,dx = -\ln\left|\cos x\right| + C$

43. $\int \cot x\,dx = \ln\left|\sin x\right| + C$

44. $\displaystyle\int_{\sin^{-1}\frac{1}{3}}^{\frac{\pi}{2}} \left(9\sin\varphi - \frac{1}{3}\csc^2\varphi\right) d\varphi = \frac{16\sqrt{2}}{3}$

Perform the indicated operations by hand. Show your work!

45. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} = \begin{pmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{pmatrix} = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}$

46. $\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \left(4+10+18\right) = \left(32\right)$

47. $\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+3y \\ 4x+5y \end{pmatrix}$

48. $\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = 3\cdot 4 - 1\cdot 2 = 10$

49. $\begin{vmatrix} 2 & 4 & 6 \\ 3 & 0 & 1 \\ 1 & 4 & 5 \end{vmatrix} = 2\begin{vmatrix} 0 & 1 \\ 4 & 5 \end{vmatrix} - 4\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} + 6\begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix} = 2(0-4) - 4(15-1) + 6(12-0)$

$= 2(-4) - 4(14) + 6(12) = -8 - 56 + 72 = 8$

50. $\begin{vmatrix} 2 & 4 & 6 \\ 7 & 8 & 9 \\ 9 & 8 & 8 \end{vmatrix} = 2\begin{vmatrix} 8 & 9 \\ 8 & 8 \end{vmatrix} - 4\begin{vmatrix} 7 & 9 \\ 9 & 8 \end{vmatrix} + 6\begin{vmatrix} 7 & 8 \\ 9 & 8 \end{vmatrix} = 2(64-72) - 4(56-81) + 6(56-72)$

$= 2(-8) - 4(-25) + 6(-16) = -16 + 100 - 96 = -12$