SECOND PARTIALS - ANSWERS

For each of the functions below, find z_{xx}, z_{xy}, z_{yx}, and z_{yy}.

1. $z = f\left(x, y\right) = x^2 + y^2$

$z_x = 2x$
$z_y = 2y$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

2. $z = f\left(x, y\right) = x^2 - y^2$

$z_x = 2x$
$z_y = -2y$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

3. $z = f\left(x, y\right) = \sqrt{xy}$

$z_x = \frac{1}{2} y(xy)^{-1/2}$

$z_y = \frac{1}{2} x(xy)^{-1/2}$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} y^2 (xy)^{-3/2} & \frac{1}{4} xy(xy)^{-3/2} \\ \frac{1}{4} xy(xy)^{-3/2} & -\frac{1}{4} x^2 (xy)^{-3/2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} y^2 (xy)^{-3/2} & \frac{1}{4} (xy)^{-1/2} \\ \frac{1}{4} (xy)^{-1/2} & -\frac{1}{4} x^2 (xy)^{-3/2} \end{pmatrix}$$

4. $z = f(x, y) = \ln(xy)$

$$z_x = \frac{1}{xy} \cdot y = \frac{1}{x}$$
$$z_y = \frac{1}{xy} \cdot x = \frac{1}{y}$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} -\frac{1}{x^2} & 0 \\ 0 & -\frac{1}{y^2} \end{pmatrix}$$

5. $z = f(x, y) = x^3 - 6x + y^3 - 9y$

$$z_x = 3x^2 - 6$$
$$z_y = 3y^2 - 9$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} 6x & 0 \\ 0 & 6y \end{pmatrix}$$

6. $z = f(x, y) = e^{5xy^2}$

$$z_x = e^{5xy^2} \cdot 5y^2 = 5y^2 e^{5xy^2}$$
$$z_y = e^{5xy^2} \cdot 10xy = 10xye^{5xy^2}$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} 25y^4 e^{5xy^2} & 5y^2 e^{5xy^2} \cdot 10xy + 10ye^{5xy^2} \\ 10xye^{5xy^2} \cdot 5y^2 + 10ye^{5xy^2} & 10xe^{5xy^2} + 100x^2y^2e^{5xy^2} \end{pmatrix}$$
$$= \begin{pmatrix} 25y^4 e^{5xy^2} & 50xy^3 e^{5xy^2} + 10ye^{5xy^2} \\ 50xy^3 e^{5xy^2} + 10ye^{5xy^2} & 10xe^{5xy^2} + 100x^2y^2e^{5xy^2} \end{pmatrix}$$

7. $z = f(x, y) = x^3y^2 + y^5$

$$z_x = 3x^2y^2$$
$$z_y = 2x^3y + 5y^4$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} 6xy^2 & 6x^2y \\ 6x^2y & 2x^3 + 20y^3 \end{pmatrix}$$

8. $z = f(x, y) = \dfrac{1}{x^2 + y^2} = (x^2 + y^2)^{-1}$

$$z_x = \frac{-2x}{(x^2 + y^2)^2}$$
$$z_y = \frac{-2y}{(x^2 + y^2)^2}$$

$$\begin{pmatrix} z_{xx} & z_{xy} \\ z_{yx} & z_{yy} \end{pmatrix} = \begin{pmatrix} \dfrac{(x^2 + y^2)^2(-2) + 4x(x^2 + y^2)(2x)}{(x^2 + y^2)^4} & \dfrac{4x(x^2 + y^2)(2y)}{(x^2 + y^2)^4} \\ \dfrac{4y(x^2 + y^2)(2x)}{(x^2 + y^2)^4} & \dfrac{(x^2 + y^2)^2(-2) + 4y(x^2 + y^2)(2y)}{(x^2 + y^2)^4} \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{(x^2 + y^2)(-2x^2 - 2y^2 + 8x^2)}{(x^2 + y^2)^4} & \dfrac{8x^3y + 8xy^3}{(x^2 + y^2)^4} \\ \dfrac{8x^3y + 8xy^3}{(x^2 + y^2)^4} & \dfrac{(x^2 + y^2)(-2x^2 - 2y^2 + 8y^2)}{(x^2 + y^2)^4} \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{6x^2 - 2y^2}{(x^2 + y^2)^3} & \dfrac{8x^3y + 8xy^3}{(x^2 + y^2)^4} \\ \dfrac{8x^3y + 8xy^3}{(x^2 + y^2)^4} & \dfrac{-2x^2 + 6y^2}{(x^2 + y^2)^3} \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{6x^2 - 2y^2}{(x^2 + y^2)^3} & \dfrac{8xy(x^2 + y^2)}{(x^2 + y^2)^4} \\ \dfrac{8xy(x^2 + y^2)}{(x^2 + y^2)^4} & \dfrac{-2x^2 + 6y^2}{(x^2 + y^2)^3} \end{pmatrix} = \begin{pmatrix} \dfrac{6x^2 - 2y^2}{(x^2 + y^2)^3} & \dfrac{8xy}{(x^2 + y^2)^3} \\ \dfrac{8xy}{(x^2 + y^2)^3} & \dfrac{-2x^2 + 6y^2}{(x^2 + y^2)^3} \end{pmatrix}$$