SECOND PARTIALS TEST

(1-7) For each of the functions below, find the critical points and the determinant of the second partials matrix, and classify each critical point as resulting in a local maximum, local minimum, saddle point, or inconclusive. Furthermore, for each critical point (a,b), specify the coordinates $\left(a,b,f(a,b)\right)$.

1. $z = f\left(x,y\right) = x^2 + y^2$

2. $z = f\left(x,y\right) = x^2 - y^2$

3. $z = f(x,y) = -\left(x^2 + y^2\right)$

4. $z = f\left(x,y\right) = x^3 - 6x + y^3 - 9y$

5. $z = f\left(x,y\right) = x^3 - 12xy - y^4$

6. $z = f\left(x,y\right) = x^3 - xy + \dfrac{y^2}{2}$

7. $z = f\left(x,y\right) = \dfrac{1}{x} + \dfrac{1}{y} + xy$

8. For a rectangular box of volume 1000 cubic feet, find the dimensions that will minimize the surface area. (Assume the box has a top, bottom, front side, back side, right side, and left side.)

9. Suppose that $y = mx + b$ is the equation for the line that best fits the points $\left(0,0\right),\left(1,3\right)$, & $\left(2,1\right)$. This means that the sum of the squares $\left(y(0)-0\right)^2 + \left(y(1)-3\right)^2 + \left(y(2)-1\right)^2$ is minimized. Find the equation for the line of best fit.

10. Find three positive numbers whose sum is 48 and whose product is as large as possible, and find that product.

11. Use the 2nd partials test to find the point in the plane $2x + y - z = -5$ that is closest to the origin. (HINT: Minimize the square of the distance from the origin. You will get the same answer, but you won't have to mess with derivatives of square roots.)

12. A company operates two plants which manufacture the same item. Suppose that the cost of operating each plant as a function of the quantities produced is $C_1 = 4q_1^2 + 10$ and $C_2 = q_2^2 + 5$. Suppose also that the total cost is $C = C_1 + C_2$, the total product demand is $q = q_1 + q_2$, and the product price as a function of demand is $p = 90 - q$. Find levels of production, q_1 and q_2, that will maximize the profit.

13. What numbers *x* and *y* come closest to satisfying the three equations $x - y = 1$, $2x + y = -1$, and $x + 2y = 1$? Solve by minimizing the sum of the squared error terms, $x - y - 1$, $2x + y + 1$, and $x + 2y - 1$.