SPHERICAL COORDINATE CONVERSIONS - ANSWERS

Convert from spherical, (ρ,θ,φ), to rectangular, (x,y,z), coordinates.

1. $(1,0,0)_{\text{spherical}} \to (0,0,1)_{\text{rectangular}}$

2. $(3,0,\pi)_{\text{spherical}} \to (0,0,-3)_{\text{rectangular}}$

3. $(1,\pi/6,\pi/6)_{\text{spherical}} \to \left(\frac{\sqrt{3}}{4},\frac{1}{4},\frac{\sqrt{3}}{2}\right)_{\text{rectangular}}$

4. $(2,\pi/2,3\pi/4)_{\text{spherical}} \to (0,\sqrt{2},-\sqrt{2})_{\text{rectangular}}$

5. $(4,\pi/4,\pi/6)_{\text{spherical}} \to (\sqrt{2},\sqrt{2},2\sqrt{3})_{\text{rectangular}}$

6. $(2,\pi/4,\pi/4)_{\text{spherical}} \to (1,1,\sqrt{2})_{\text{rectangular}}$

Convert from rectangular, (x,y,z), to spherical, (ρ,θ,φ), coordinates.

7. $(-3,0,0)_{\text{rectangular}} \to \left(3,\pi,\frac{\pi}{2}\right)_{\text{spherical}}$

8. $(1,1,\sqrt{2})_{\text{rectangular}} \to \left(2,\frac{\pi}{4},\frac{\pi}{4}\right)_{\text{spherical}}$

9. $(\sqrt{3},0,1)_{\text{rectangular}} \to \left(2,0,\frac{\pi}{3}\right)_{\text{spherical}}$

10. $(-\sqrt{3},-3,-2)_{\text{rectangular}} \to \left(4,\frac{4\pi}{3},\frac{2\pi}{3}\right)_{\text{spherical}}$

11. $(1,-1,-\sqrt{2})_{\text{rectangular}} \to \left(2,\frac{7\pi}{4},\frac{3\pi}{4}\right)_{\text{spherical}}$

12. $(\sqrt{3},1,2\sqrt{3})_{\text{rectangular}} \to \left(4,\frac{\pi}{6},\frac{\pi}{6}\right)_{\text{spherical}}$

Write the given equation in spherical coordinates.

13. $x^2 + y^2 + z^2 = 25$

$\rho^2 = 25 \Rightarrow \rho = 5$

14. $x^2 + y^2 = 2y$

$\rho^2 \sin^2\varphi\cos^2\theta + \rho^2 \sin^2\varphi\sin^2\theta = 2\rho\sin\varphi\sin\theta$
$\Rightarrow \rho^2 \sin^2\varphi(\cos^2\theta + \sin^2\theta) = 2\rho\sin\varphi\sin\theta$
$\Rightarrow \rho^2 \sin^2\varphi = 2\rho\sin\varphi\sin\theta$
$\Rightarrow \rho^2 \sin^2\varphi - 2\rho\sin\varphi\sin\theta = 0$
$\Rightarrow \rho\sin\varphi(\rho\sin\varphi - 2\sin\theta) = 0$

15. $x^2 + y^2 + 9z^2 = 36$

$\rho^2 \sin^2\varphi\cos^2\theta + \rho^2 \sin^2\varphi\sin^2\theta + 9\rho^2\cos^2\varphi = 36$
$\Rightarrow \rho^2 \sin^2\varphi(\cos^2\theta + \sin^2\theta) + 9\rho^2\cos^2\varphi = 36$
$\Rightarrow \rho^2 \sin^2\varphi + 9\rho^2\cos^2\varphi = 36$
$\Rightarrow \rho^2(\sin^2\varphi + 9\cos^2\varphi) = 36$
$\Rightarrow \rho^2(\sin^2\varphi + \cos^2\varphi + 8\cos^2\varphi) = 36$
$\Rightarrow \rho^2(1 + 8\cos^2\varphi) = 36$

16. $z = 1$ (write as a function of ρ)

$\rho\cos\varphi = 1 \Rightarrow \rho = \sec\varphi$