

SPHERICAL INTEGRALS

For each problem below, set up and evaluate a triple integral in spherical coordinates.

1. Let V be a sphere with center at the origin and radius $= r$. Find the volume of V .
2. Find the volume of the solid bounded above by the sphere $x^2 + y^2 + z^2 = 1$ and below by the cone $z = \sqrt{x^2 + y^2}$.
3. Evaluate $\iiint_V \frac{1}{x^2 + y^2 + z^2} dV$ where V is the solid region between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$.
4. Find the volume of the solid bounded above by $z = 1$ and below by $z = \sqrt{x^2 + y^2}$.
5. Suppose you drill a hole of radius 1 through the center of a sphere of radius 3. Find the volume of the portion removed by the drill.
6. Find the volume of the solid region defined by $\rho = \sin \varphi$ where $0 \leq \theta \leq 2\pi$ and $0 \leq \varphi \leq \pi$.
7. An object occupies the region inside the unit sphere with center at the origin and has density equal to the square of the distance from the origin. Find the mass.