SURFACE AREA - ANSWERS

(1-5) Use the formula $\text{Surface Area } = \iint_S dS = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$ to find the surface area of the following planes over the region defined, for problems 1 through 3, by the intervals $0 \le x \le 1$ and $0 \le y \le 1$, and, for problems 4 and 5, by the intervals $0 \le x \le 2$ and $0 \le y \le 2$.

1. $z = x + y + 3$

$$\iint_S dS = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA = \int_0^1 \int_0^1 \sqrt{1+1+1}\, dydx = \sqrt{3}$$

2. $z = 2x - y + 1$

$$\iint_S dS = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA = \int_0^1 \int_0^1 \sqrt{4+1+1}\, dydx = \sqrt{6}$$

3. $z = 3x + 2y + 4$

$$\iint_S dS = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA = \int_0^1 \int_0^1 \sqrt{9+4+1}\, dydx = \sqrt{14}$$

4. $z = 8x + 4y + 2$

$$\iint_S dS = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA = \int_0^2 \int_0^2 \sqrt{64+16+1}\, dydx = 4\sqrt{81} = 36$$

5. $z = -x - y - 10$

$$\iint_S dS = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA = \int_0^2 \int_0^2 \sqrt{1+1+1}\, dydx = 4\sqrt{3}$$

6. Find the area of the portion of the plane $x + y + z = 1$ in the first octant.

Let $z = -x - y + 1$. Then,

$$\iint_S dS = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA = \int_0^1 \int_0^{-x+1} \sqrt{1+1+1}\, dy dx = \sqrt{3} \int_0^1 \int_0^{-x+1} dy dx = \frac{\sqrt{3}}{2}.$$