SURFACE INTEGRALS

In each problem below you are given a surface *S*, defined by $z = f(x, y)$, over a region *R*, defined by the given limits on *x* and *y*. You are also given a function $g = g(x, y, z)$. Use the relation $dS = \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA = \sqrt{z_x{}^2 + z_y{}^2 + 1}\, dA$ to find the value of the surface integral of *g* on the surface *S* by evaluating

$$\text{Surface Integral} = \iint_S g(x, y, z)\, dS = \iint_R g(x, y, z) \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

$$= \iint_R g(x, y, z) \sqrt{z_x{}^2 + z_y{}^2 + 1}\, dA.$$

$S : z = y$

1. $R : 0 \le x \le 1,\ 0 \le y \le 2$
   $g(x, y, z) = x + y + z$

$S : z = x^2 + y^2 + 1$

2. $R : 0 \le x \le 1,\ 0 \le y \le 1$
   $g(x, y, z) = \dfrac{z}{\sqrt{4x^2 + 4y^2 + 1}}$

$S : z = -x - y + 2$

3. $R : 0 \le x \le \dfrac{\pi}{2},\ 0 \le y \le \dfrac{\pi}{2}$
   $g(x, y, z) = \cos x + \sin y$