TOTAL DIFFERENTIAL - ANSWERS

For each of the following functions, find the total differential.

1. $z = f(x,y) = x^3y^2$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = 3x^2y^2dx + 2x^3ydy$$

2. $z = f(x,y) = \sin(x^3y^2)$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \cos(x^3y^2)\cdot 3x^2y^2dx + \cos(x^3y^2)\cdot 2x^3ydy$$
$$= 3x^2y^2\cos(x^3y^2)dx + 2x^3y\cos(x^3y^2)dy$$

3. $z = f(x,y) = \sqrt{x^3y^2}$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{1}{2\sqrt{x^3y^2}}\cdot 3x^2y^2dx + \frac{1}{2\sqrt{x^3y^2}}\cdot 2x^3ydy$$
$$= \frac{3x^2y^2}{2\sqrt{x^3y^2}}dx + \frac{x^3y}{\sqrt{x^3y^2}}dy$$

4. $z = f(x,y) = \sec(x^3y^2)$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \sec(x^3y^2)\tan(x^3y^2)\cdot 3x^2y^2dx + \sec(x^3y^2)\tan(x^3y^2)\cdot 2x^3ydy$$
$$= 3x^2y^2\sec(x^3y^2)\tan(x^3y^2)dx + 2x^3y\sec(x^3y^2)\tan(x^3y^2)dy$$

5. $z = f(x,y) = \tan(x^3y^2)$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \sec^2(x^3y^2)\cdot 3x^2y^2dx + \sec^2(x^3y^2)\cdot 2x^3ydy$$
$$= 3x^2y^2\sec^2(x^3y^2)dx + 2x^3y\sec^2(x^3y^2)dy$$

6. $z = f(x,y) = \sin^{-1}(x^3y^2)$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{1}{\sqrt{1-(x^3y^2)^2}} \cdot 3x^2y^2dx + \frac{1}{\sqrt{1-(x^3y^2)^2}} \cdot 2x^3ydy$$
$$= \frac{3x^2y^2}{\sqrt{1-(x^3y^2)^2}}dx + \frac{2x^3y}{\sqrt{1-(x^3y^2)^2}}dy$$

7. $z = f(x,y) = \sqrt[3]{x^2+y+4}$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{1}{3}(x^2+y+4)^{-2/3}(2x)dx + \frac{1}{3}(x^2+y+4)^{-2/3}dy$$
$$= \frac{2x}{3(x^2+y+4)^{2/3}}dx + \frac{1}{3(x^2+y+4)^{2/3}}dy$$

8. $z = f(x,y) = e^{-(x^2+y^2)}$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = e^{-(x^2+y^2)}(-2x)dx + e^{-(x^2+y^2)}(-2y)dy$$
$$= \frac{-2x}{e^{(x^2+y^2)}}dx + \frac{-2y}{e^{(x^2+y^2)}}dy$$

9. $z = f(x,y) = \ln(xy)$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{1}{x}dx + \frac{1}{y}dy$$

10. $z = f(x,y) = \frac{xy+1}{x+y}$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{y^2-1}{(x+y)^2}dx + \frac{x^2-1}{(x+y)^2}dy$$