TOTAL DIFFERENTIAL APPROXIMATIONS - ANSWERS

For each of the following functions, use the value $f(1,2)$ and the total differential to approximate $f(1.01,2.03)$ and Δz rounded to four decimal places. Let $\Delta x = 0.01$ and $\Delta y = 0.03$. Additionally, also use your calculator to compute $f(1.01,2.03)$ rounded to four decimal places.

1. $z = f(x,y) = x^3y^2$

$f(1,2) = 4$, $\Delta x = 0.01$, and $\Delta y = 0.03$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = 3x^2y^2dx + 2x^3ydy$$

$$f(1.01,2.03) \approx \left.\frac{\partial f}{\partial x}\right|_{(1,2)} \Delta x + \left.\frac{\partial f}{\partial y}\right|_{(1,2)} \Delta y + f(1,2) = (12)(0.01) + (4)(0.03) + 4 = 4.24$$

$f(1.01,2.03) \approx 4.2458$

2. $z = f(x,y) = \sin(x^3y^2)$

$f(1,2) = \sin(4)$, $\Delta x = 0.01$, and $\Delta y = 0.03$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \cos(x^3y^2)\cdot 3x^2y^2dx + \cos(x^3y^2)\cdot 2x^3ydy$$

$$= 3x^2y^2\cos(x^3y^2)dx + 2x^3y\cos(x^3y^2)dy$$

$$f(1.01,2.03) \approx \left.\frac{\partial f}{\partial x}\right|_{(1,2)} \Delta x + \left.\frac{\partial f}{\partial y}\right|_{(1,2)} \Delta y + f(1,2) = (12)\cos(4)(0.01) + (4)\cos(4)(0.03) + \sin(4) \approx -0.9137$$

$f(1.01,2.03) \approx -0.8931$

3. $z = f(x,y) = \sqrt{x^3y^2}$

$f(1,2) = 2$, $\Delta x = 0.01$, and $\Delta y = 0.03$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{1}{2\sqrt{x^3y^2}}\cdot 3x^2y^2dx + \frac{1}{2\sqrt{x^3y^2}}\cdot 2x^3ydy$$

$$= \frac{3x^2y^2}{2\sqrt{x^3y^2}}dx + \frac{x^3y}{\sqrt{x^3y^2}}dy$$

$$f(1.01,2.03) \approx \left.\frac{\partial f}{\partial x}\right|_{(1,2)} \Delta x + \left.\frac{\partial f}{\partial y}\right|_{(1,2)} \Delta y + f(1,2) = (3)(0.01) + (1)(0.03) + 2 = 2.06$$

$f(1.01,2.03) \approx 2.0605$

4. $z = f(x,y) = \sec(x^3y^2)$

$f(1,2) = \sec(4)$, $\Delta x = 0.01$, and $\Delta y = 0.03$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \sec(x^3y^2)\tan(x^3y^2)\cdot 3x^2y^2dx + \sec(x^3y^2)\tan(x^3y^2)\cdot 2x^3ydy$$

$$= 3x^2y^2\sec(x^3y^2)\tan(x^3y^2)dx + 2x^3y\sec(x^3y^2)\tan(x^3y^2)dy$$

$$f(1.01, 2.03) \approx \left.\frac{\partial f}{\partial x}\right|_{(1,2)}\Delta x + \left.\frac{\partial f}{\partial y}\right|_{(1,2)}\Delta y + f(1,2)$$

$$= \sec(4)\tan(4)(12)(0.01) + \sec(4)\tan(4)(4)(0.03) + \sec(4) \approx -1.9550$$

$f(1.01, 2.03) \approx -2.2229$

5. $z = f(x,y) = \tan(x^3y^2)$

$f(1,2) = \tan(4)$, $\Delta x = 0.01$, and $\Delta y = 0.03$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \sec^2(x^3y^2)\cdot 3x^2y^2dx + \sec^2(x^3y^2)\cdot 2x^3ydy$$

$$= 3x^2y^2\sec^2(x^3y^2)dx + 2x^3y\sec^2(x^3y^2)dy$$

$$f(1.01, 2.03) \approx \left.\frac{\partial f}{\partial x}\right|_{(1,2)}\Delta x + \left.\frac{\partial f}{\partial y}\right|_{(1,2)}\Delta y + f(1,2)$$

$$= \sec^2(4)(12)(0.01) + \sec^4(4)(4)(0.03) + \tan(4) \approx 1.7196$$

$f(1.01, 2.03) \approx 1.9852$

6. $z = f(x,y) = \sin^{-1}(x^3y^2)$

$f(1,2) = \sin^{-1}(4)$, $\Delta x = 0.01$, and $\Delta y = 0.03$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{1}{\sqrt{1-(x^3y^2)^2}}\cdot 3x^2y^2dx + \frac{1}{\sqrt{1-(x^3y^2)^2}}\cdot 2x^3ydy$$

$$= \frac{3x^2y^2}{\sqrt{1-(x^3y^2)^2}}dx + \frac{2x^3}{\sqrt{1-(x^3y^2)^2}}dy$$

$$f(1.01, 2.03) \approx \left.\frac{\partial f}{\partial x}\right|_{(1,2)}\Delta x + \left.\frac{\partial f}{\partial y}\right|_{(1,2)}\Delta y + f(1,2) = \text{ domain error}$$

$f(1.01, 2.03) \approx$ domain error

The point $(1,2)$ is outside of the domain of the function.