WHAT ARE THEY GOOD FOR?

Determinants of Matrices

Used to compute cross products
Used to compute Jacobians in change of variables for multiple integrals

Cylindrical and Spherical Coordinates

Simplifying integrals by changing to a different coordinate system

Parametrization of Curves

Parametrizing curves in order to easily evaluate line integrals

Norm of a Vector

Used in calculating unit vectors
Used in calculating arc length
Used in calculating curvature
Used in calculating certain line integrals

Unit Vectors

Calculating the unit tangent vector
Calculating the unit normal vector
If $\vec{u}$ is a unit vector and $\vec{F}$ is a force, then $\vec{F} \bullet \vec{u}$ gives the component of $\vec{F}$ in the direction of $\vec{u}$.

Dot Product

Angle between two vectors
Determining if two vectors are perpendicular
Square of the length of a vector
Work done by a force in moving an object a given distance
Used to show that the coefficients of the equation for a plane define a vector perpendicular to the plane

Cross Product

Finding a vector perpendicular to two other vectors
Finding a plane containing two vectors and a given point
Finding the area of a parallelogram

Finding the Area of a Parallelogram

This formula is used to help develop integration formulas for surface integrals.
Used also to develop the formula for doing a general change of variables in multiple integration.

Derivatives of Vector-valued Functions

Used in arc length formula
Used in finding unit tangent and unit normal vectors
Used in formulas for line integrals

Unit Tangent and Normal Vectors

$\vec{F} \bullet T$ gives the component of force in the direction of T, and this can be used in a line integral to compute either the work or circulation created by a force along a particular path.
$\vec{F} \bullet N$ gives the component of force in the direction of N, and this can be used in a line integral to compute the flux of material pushed across a boundary by the force.

Arc Length

Finding the length of a curve
Using ds/dt to find curvature
Using the differential version of ds/dt to evaluate line integrals with respect to ds, arc length

Curvature

We won't see curvature again, but the $ds/dt = \|r'(t)\|$ formula that was used in calculating curvature will be used frequently once we get to line integrals.

Partial Derivatives

Finding rates of change in the directions of positive *x* and *y*
Constructing tangent planes
Used in optimization problems
Used in constrained optimization problems

Total Differential

Estimate change in *z*
Provides easy derivation of the chain rule

Chain Rule

Often used in the proof of key theorems in multivariable calculus

Gradient

Used in computing directional derivatives
Shows the direction and maximum value of rate of change
Shows the direction and minimum value of rate of change
Used in proof of theorem on Lagrange multipliers

Multiple Integration

Used for finding volumes, probabilities, areas, and masses (among other things)

Vector Fields

Used to model the action of forces in 2 and 3-dimensional space

Curl

Used with line integrals to compute flow and circulation along a path created by a vector field

Divergence

Used with line integrals to compute the flux of material pushed across a boundary by a vector field

Line Integrals

Used to define extensions of the Riemann integral
Used in computing work done by a vector field in moving a particle along a path
Used in computing flow and circulation along a path created by a vector field
Used in computing the flux of material pushed across a boundary by a vector field