

WORK - ANSWERS

Determine the work, $\int_C \vec{F} \cdot d\vec{r}$, done by each vector field below along the indicated path.

1. $\vec{F} = x\hat{i} + y\hat{j}$ and C is the line segment from $(1,2)$ to $(5,10)$.

$$\vec{F} = x\hat{i} + y\hat{j}, P = x, Q = y$$

$$\begin{aligned} x &= 1 + 4t & \frac{dx}{dt} &= 4 \\ y &= 2 + 8t \Rightarrow & & \\ 0 \leq t \leq 1 & & \frac{dy}{dt} &= 8 \end{aligned}$$

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_C P dx + Q dy = \int_a^b \left(P \frac{dx}{dt} + Q \frac{dy}{dt} \right) dt = \int_0^1 [(1+4t)4 + (2+8t)8] dt \\ &= \int_0^1 (20 + 80t) dt = 20t + 40t^2 \Big|_0^1 = 60 \end{aligned}$$

2. $\vec{F} = x\hat{i} + y\hat{j}$ and C is the line segment from $(5,10)$ to $(1,2)$.

$$\vec{F} = x\hat{i} + y\hat{j}, P = x, Q = y$$

$$\begin{aligned} x &= 5 - 4t & \frac{dx}{dt} &= -4 \\ y &= 10 - 8t \Rightarrow & & \\ 0 \leq t \leq 1 & & \frac{dy}{dt} &= -8 \end{aligned}$$

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_C P dx + Q dy = \int_a^b \left(P \frac{dx}{dt} + Q \frac{dy}{dt} \right) dt = \int_0^1 [(5-4t)(-4) + (10-8t)(-8)] dt \\ &= \int_0^1 (-100 + 80t) dt = -100t + 40t^2 \Big|_0^1 = -60 \end{aligned}$$

3. $\vec{F} = -x\hat{i} - y\hat{j}$ and C is the line segment from $(1,2)$ to $(5,10)$.

$$\vec{F} = x\hat{i} + y\hat{j}, P = -x, Q = -y$$

$$\begin{aligned}x &= 1 + 4t & \frac{dx}{dt} &= 4 \\y &= 2 + 8t \Rightarrow & \frac{dy}{dt} &= 8 \\0 &\leq t \leq 1\end{aligned}$$

$$\begin{aligned}\int_C \vec{F} \cdot d\vec{r} &= \int_C P dx + Q dy = \int_a^b \left(P \frac{dx}{dt} + Q \frac{dy}{dt} \right) dt = \int_0^1 [(-1-4t)4 + (-2-8t)8] dt \\&= \int_0^1 (-20 - 80t) dt = -20t - 40t^2 \Big|_0^1 = -60\end{aligned}$$

4. $\vec{F} = x\hat{i} + y\hat{j}$ and C is the unit circle oriented counterclockwise.

$$\vec{F} = x\hat{i} + y\hat{j}, P = x, Q = y$$

$$\begin{aligned}x &= \cos t & \frac{dx}{dt} &= -\sin t \\y &= \sin t \Rightarrow & \frac{dy}{dt} &= \cos t \\0 &\leq t \leq 2\pi\end{aligned}$$

$$\begin{aligned}\int_C \vec{F} \cdot d\vec{r} &= \int_C P dx + Q dy = \int_a^b \left(P \frac{dx}{dt} + Q \frac{dy}{dt} \right) dt = \int_0^{2\pi} [\cos t(-\sin t) + \sin t \cos t] dt \\&= \int_0^{2\pi} 0 dt = 0\end{aligned}$$

5. $\vec{F} = -y\hat{i} + x\hat{j}$ and C is the unit circle oriented counterclockwise.

$$\vec{F} = -y\hat{i} + x\hat{j}, P = -y, Q = x$$

$$\begin{aligned} x = \cos t & \quad \frac{dx}{dt} = -\sin t \\ y = \sin t & \Rightarrow \\ 0 \leq t \leq 2\pi & \quad \frac{dy}{dt} = \cos t \end{aligned}$$

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_C P dx + Q dy = \int_a^b \left(P \frac{dx}{dt} + Q \frac{dy}{dt} \right) dt = \int_0^{2\pi} [(-\sin t)(-\sin t) + \cos t \cos t] dt \\ &= \int_0^{2\pi} 1 dt = 2\pi \end{aligned}$$

6. $\vec{F} = -y\hat{i} + x\hat{j}$ and C is the unit circle oriented clockwise.

$$\vec{F} = -y\hat{i} + x\hat{j}, P = -y, Q = x$$

$$\begin{aligned} x = \cos t & \quad \frac{dx}{dt} = -\sin t \\ y = -\sin t & \Rightarrow \\ 0 \leq t \leq 2\pi & \quad \frac{dy}{dt} = -\cos t \end{aligned}$$

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_C P dx + Q dy = \int_a^b \left(P \frac{dx}{dt} + Q \frac{dy}{dt} \right) dt = \int_0^{2\pi} [(\sin t)(-\sin t) + \cos t(-\cos t)] dt \\ &= \int_0^{2\pi} (-\sin^2 t - \cos^2 t) dt = -\int_0^{2\pi} dt = -2\pi \end{aligned}$$

7. $\vec{F} = y\hat{i} - x\hat{j}$ and C is the unit circle oriented counterclockwise.

$$\vec{F} = y\hat{i} - x\hat{j}, P = y, Q = -x$$

$$\begin{aligned} x = \cos t & \quad \frac{dx}{dt} = -\sin t \\ y = \sin t & \Rightarrow \\ 0 \leq t \leq 2\pi & \quad \frac{dy}{dt} = \cos t \end{aligned}$$

$$\begin{aligned} \int_C \vec{F} \cdot d\vec{r} &= \int_C P dx + Q dy = \int_a^b \left(P \frac{dx}{dt} + Q \frac{dy}{dt} \right) dt = \int_0^{2\pi} [(\sin t)(-\sin t) - \cos t(\cos t)] dt \\ &= \int_0^{2\pi} (-\sin^2 t - \cos^2 t) dt = -\int_0^{2\pi} dt = -2\pi \end{aligned}$$