

c. Compute $\Gamma(1)$ by the definition.

Definition: The Gamma Function is defined by $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$, $x > 0$.

$$\text{Therefore, } \Gamma(1) = \int_0^{\infty} t^{1-1} e^{-t} dt = \int_0^{\infty} e^{-t} dt = \left. \frac{-1}{e^t} \right|_0^{\infty} = \lim_{n \rightarrow \infty} \left. \frac{-1}{e^t} \right|_0^n = \lim_{n \rightarrow \infty} \left(\frac{-1}{e^n} + \frac{1}{e^0} \right) = 1.$$