d. Compute $\Gamma(2), \Gamma(3), \Gamma(4),$ and $\Gamma(5)$ using the previous two results.

<u>Definition:</u> The Gamma Function is defined by $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t}\, dt$, $x > 0$.

Therefore,

$$\Gamma(1) = \int_0^\infty t^{1-1} e^{-t}\, dt = \int_0^\infty e^{-t}\, dt = \left.\frac{-1}{e^t}\right|_0^\infty = \lim_{n\to\infty} \left.\frac{-1}{e^t}\right|_0^n = \lim_{n\to\infty}\left(\frac{-1}{e^n} + \frac{1}{e^0}\right) = 1.$$

Also, for $x > 0$, $\Gamma(x+1) = x\Gamma(x)$.

Hence,

$$\Gamma(2) = \Gamma(1+1) = 1 \cdot \Gamma(1) = 1$$
$$\Gamma(3) = \Gamma(2+1) = 2 \cdot \Gamma(2) = 2 \cdot 1 = 2$$
$$\Gamma(4) = \Gamma(3+1) = 3 \cdot \Gamma(3) = 3 \cdot 2 \cdot 1 = 6$$
$$\Gamma(5) = \Gamma(4+1) = 4 \cdot \Gamma(4) = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$