d2. <u>Prove:</u> If n is a positive integer, then $\Gamma(n+1)=n!$.

<u>Proof:</u> We begin by verifying that

$\Gamma(1)=\int_0^\infty t^{1-1}e^{-t}\,dt=\int_0^\infty e^{-t}\,dt=\left.\frac{-1}{e^t}\right|_0^\infty=\lim_{n\to\infty}\left.\frac{-1}{e^t}\right|_0^n=\lim_{n\to\infty}\left(\frac{-1}{e^n}+\frac{1}{e^0}\right)=1=1!$. Now suppose that the result is true for some positive integer n, i.e. $\Gamma(n)=(n-1)!$. Then $\Gamma(n+1)=n\cdot\Gamma(n)=n\cdot(n-1)!=n!$. Therefore, by mathematical induction, the result is true for all positive integers n.