

f. Compute $\Gamma\left(-\frac{1}{2}\right)$ and $\Gamma\left(\frac{3}{2}\right)$.

We can use the formula $\Gamma(x+1) = x\Gamma(x)$, for $x > 0$, to extend the definition of the Gamma Function to most negative numbers. In particular,

$$\sqrt{\pi} = \Gamma\left(\frac{1}{2}\right) = \Gamma\left(-\frac{1}{2} + 1\right) = -\frac{1}{2}\Gamma\left(-\frac{1}{2}\right) \Rightarrow \Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}.$$

Also,

$$\Gamma\left(\frac{3}{2}\right) = \Gamma\left(\frac{1}{2} + 1\right) = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{1}{2}\sqrt{\pi} = \frac{\sqrt{\pi}}{2}.$$