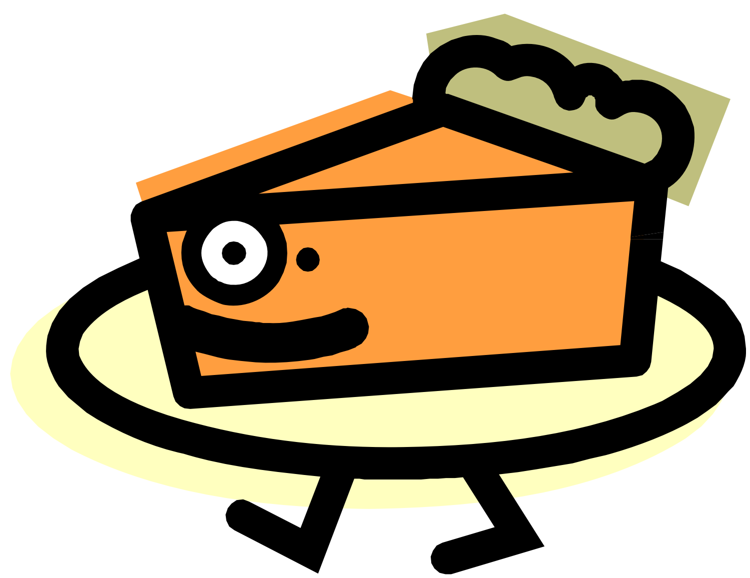
π Day



π

$$\pi$$

Why is this number different from all other numbers?

$$\pi$$

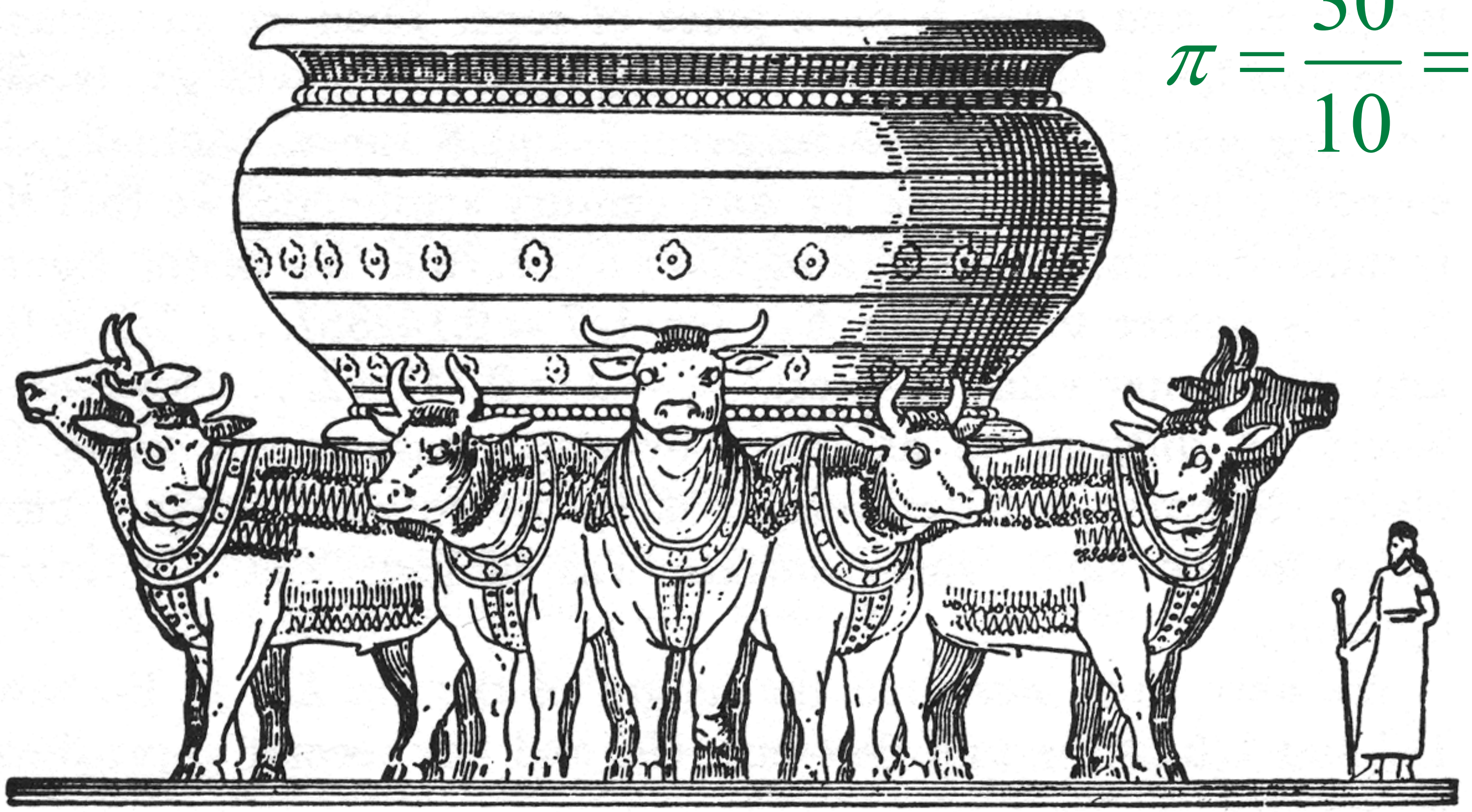
Why is this number different from all other numbers?

We begin to explain …

A verse in the Bible seems to give an exact value of “3” for pi.

“He made the sea of a casting ten cubits from one lip to the other lip, circular all around, five cubits its height, a measuring line thirty cubits could encircle it all around.”
–I Kings 7:23 (circa 550 BCE)

$$\pi = \frac{30}{10} = 3$$



However, in Hebrew, each letter is also a number, and an odd spelling of the word for “measuring line” seems to indicate how to get a value for pi that is accurate to four decimal places.

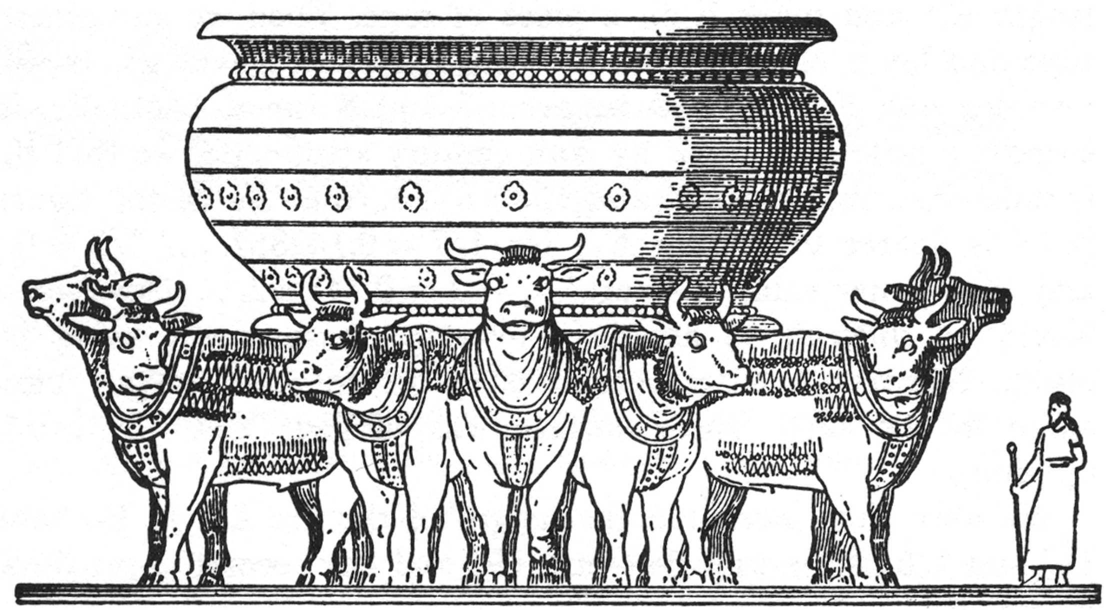
measuring line thirty cubits

קוה שלשים באמה

incorrect → קוה = 111

correct → קו = 106

$$\pi = \frac{30}{10} \cdot \frac{111}{106} = 3.141509434\ldots$$



Additionally, there is an extraordinary “Bible code” where the digits 3,1,4,1,5 appear in this verse from First Kings beginning in the middle of the word for “circular.” The length of the skip sequence for this code is 8,976 letters.

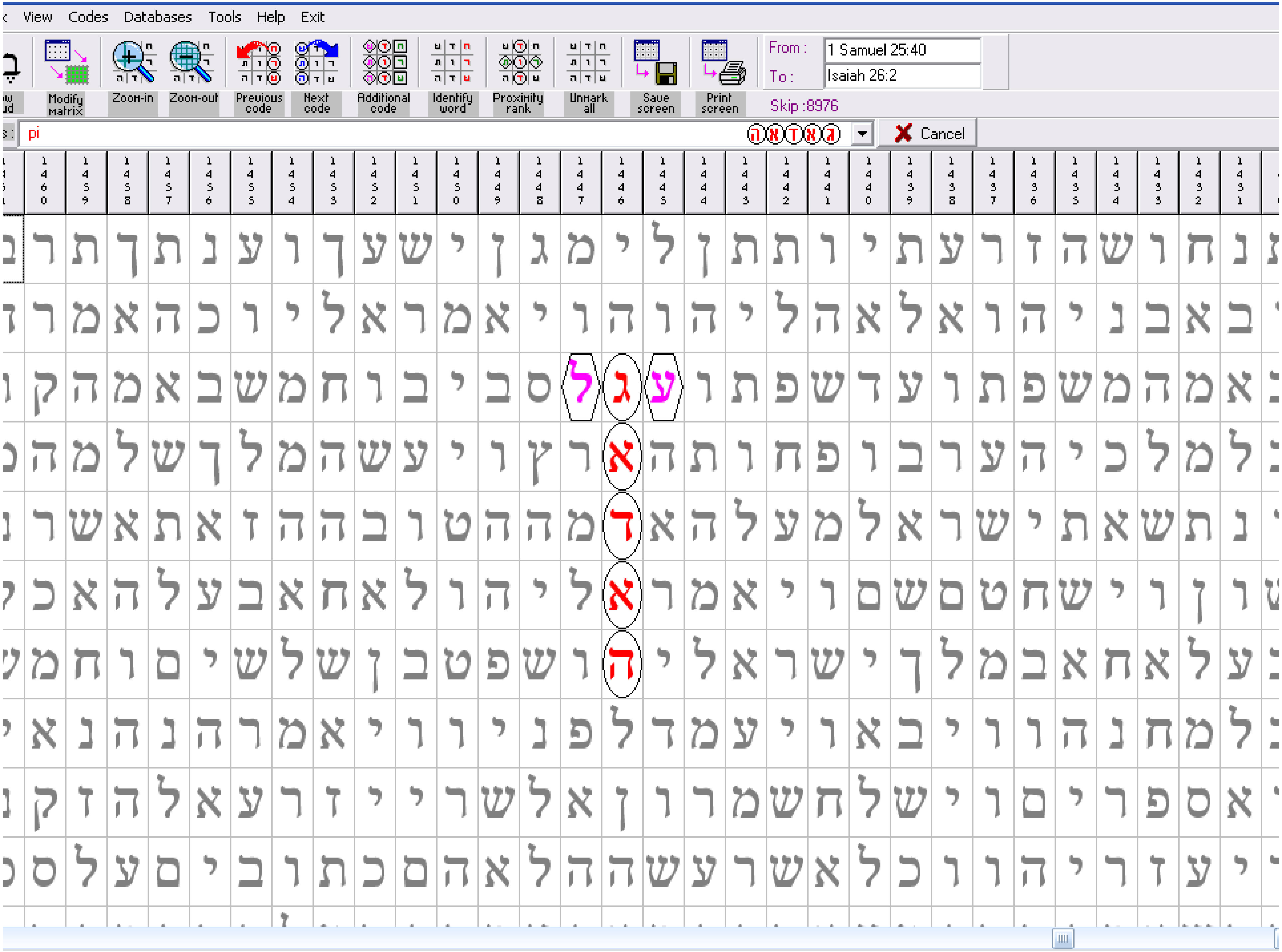
ג → 3

א → 1

ד → 4

א → 1

ה → 5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Save screen
Print screen
From :
1 Samuel 25:40
To :
Isaiah 26:2
Skip :8976
pi
Cancel

Here are some early estimates of pi and the dudes that did them.

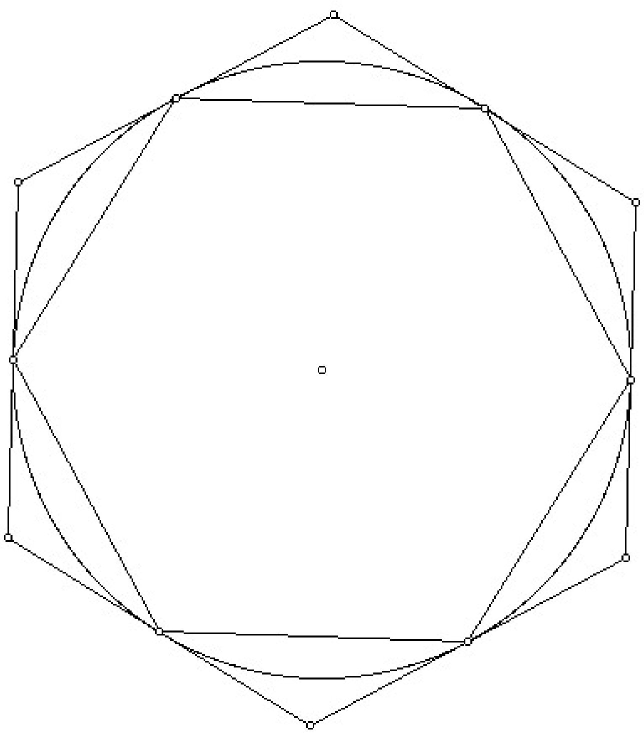
The Rhind Papyrus

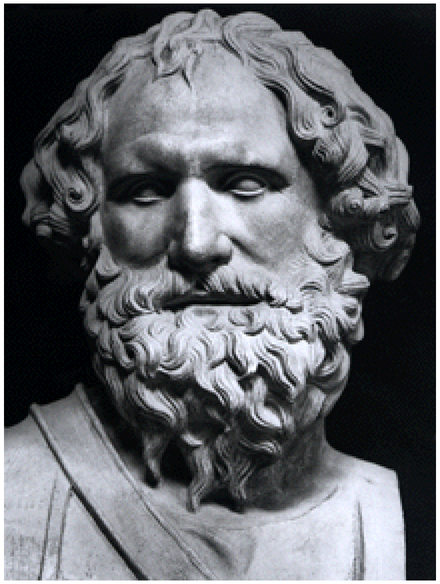
“Cut off 1/9 of a diameter and construct a square of the remainder. This has the same area as the circle.”

-Ahmes the scribe
1650 BCE

$$\pi = \frac{256}{81} \approx 3.160493827$$

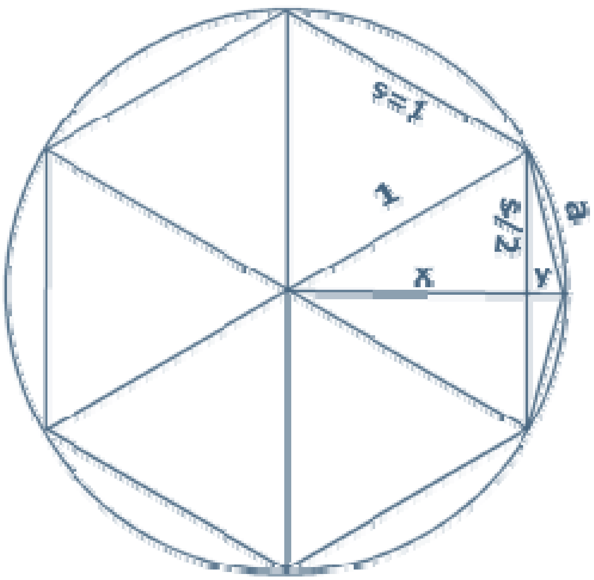
Archimedes
287 BCE – 212 BCE





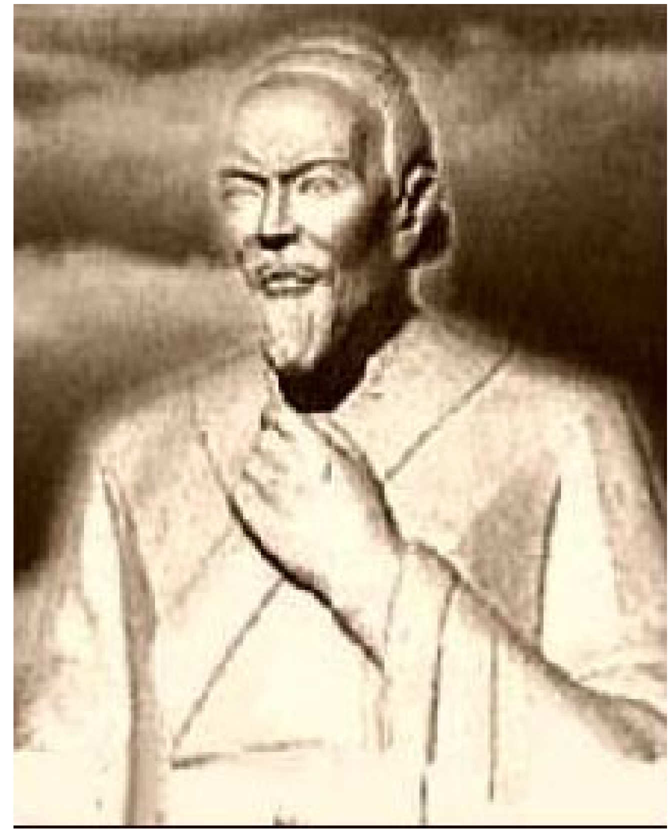
“The ratio of the circumference of any circle to its diameter is less than 3 1/7, but greater than 3 10/71.”

Claudius Ptolemy
90 – 168 CE



$$\pi = 3^{\circ}\, 8'\, 30'' = 3 + \frac{8}{60} + \frac{30}{3600} = 3.141\overline{6}$$

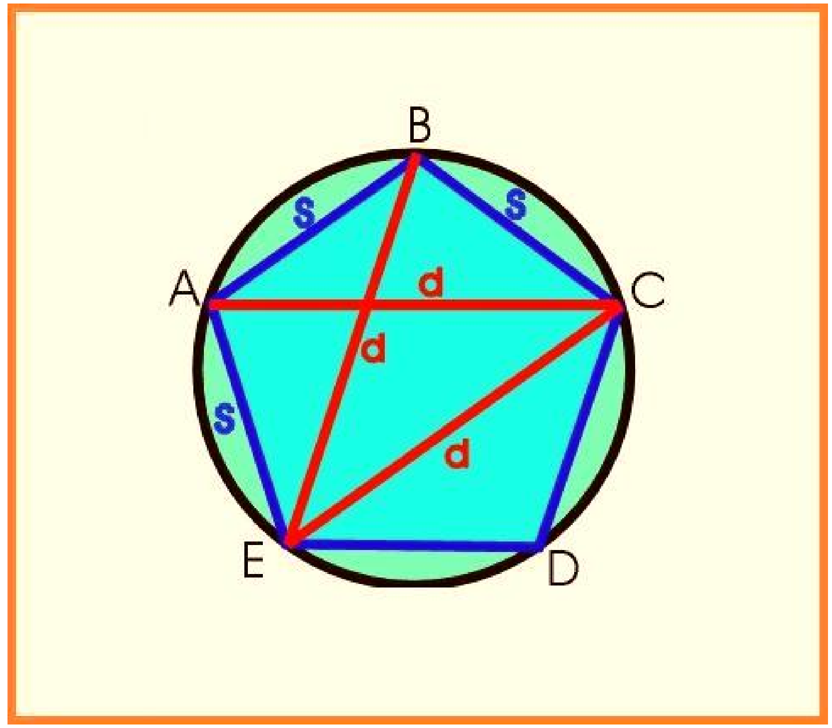
Tsu Ch'ung Chih
b. 429 CE



$$\pi = \frac{355}{113} = 113\overset{3.14159292...}{\overline{)355}}$$

This is my favorite! Easy to remember using the odd numbers, 1, 3, 5.

Brahmagupta
598 – 668 CE



$$\pi \approx \sqrt{9.65}, \sqrt{9.81}, \sqrt{9.86}, \sqrt{9.87}, \ldots$$

$$\pi \overset{?}{=} \sqrt{10} = 3.16\ldots \quad \text{(close, but no cigar!)}$$

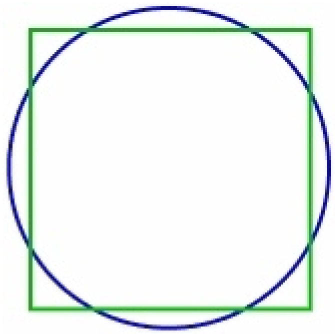
In 1761, Lambert proved the pi is irrational. I am also irrational.

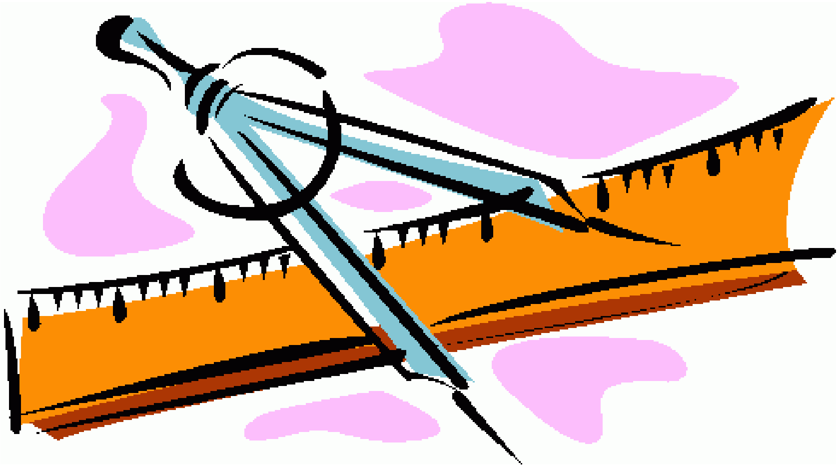
Johann Heinrich Lambert
1728 – 1777 CE

- If x is rational, then *tan(x)* is irrational
- *tan*(π/4) = 1
- Therefore, π/4 is irrational
- Therefore, π is irrational (1761)

Pi is also related to the problem of “squaring the circle.”

Squaring the Circle





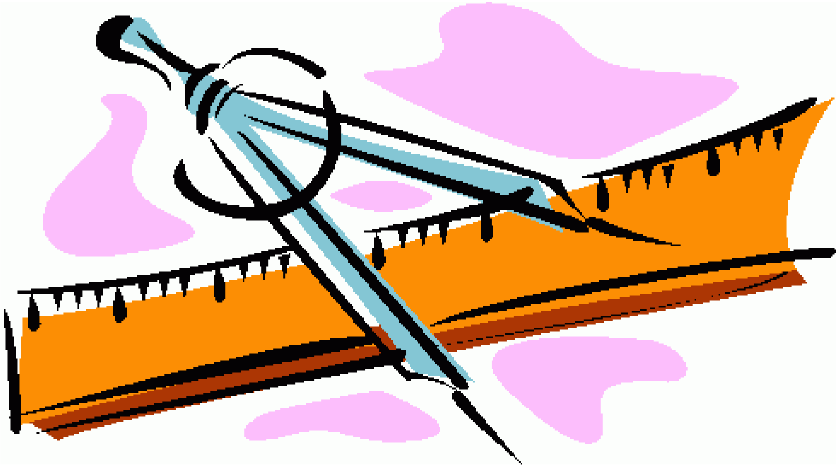
❖Using a compass and straightedge, construct a square with the same area as a circle

❖Equivalent to constructing a line of length π

Euclid
circa 300 BCE

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. Given any straight line and a point not on it, there "exists one and only one straight line which passes" through that point and never intersects the first line, no matter how far they are extended.

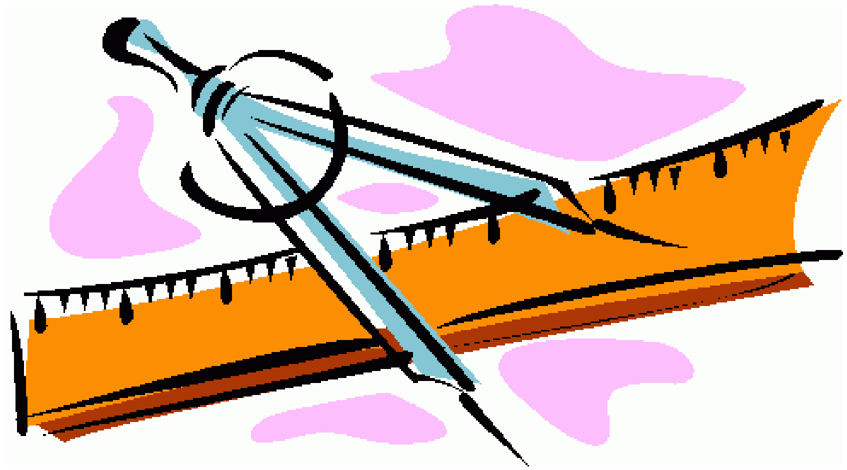
Compass & Ruler Constructions



- ❖Euclid's Postulates →
- ❖Compass &Ruler Constructions →
- ❖First and Second Degree Polynomials →
- ❖Polynomial Equations with Integer Coefficients →
- ❖Algebraic Numbers →
- ❖If you can square the circle, then π is algebraic

Ferdinand von Lindemann proved that pi is “transcendental.” I am also transcendental.

Ferdinand von Lindemann
1852 – 1939



- If x is algebraic, then e^x is transcendental (i.e. not algebraic)
- If π is algebraic, then so is $i\pi$
- $e^{i\pi} = -1$, an algebraic number
- Therefore, π is transcendental (1882)
- Therefore, you can't square the circle

Ferdinand von Lindemann’s tombstone shows pi inside a square.


π
Universitätsprofessor
Dr. Ferdinand von Lindemann
* 12. April 1852 † 6. März 1939
Schriftstellerin
Lisbeth von Lindemann
geborene Küssner
* 22. Juli 1861 † 28. Febr. 1936

By the way, I'm mathematically descended from Ferdinand von Lindemann. You just go back from me to my advisor in graduate school, then back to her advisor, and so on.

THE MATHEMATICS GENEALOGY PROJECT

Gottfried Leibniz → Jacob Bernoulli → Johann Bernoulli →
Leonhard Euler → Joseph Lagrange
→ Jean-Baptiste Joseph Fourier, Simeon Poisson
→ Gustav Dirichlet
→ Rudolph Lipschitz → Felix Klein
→ C. L. Ferdinand Lindemann
→ David Hilbert → Hellmuth Kneser
→ Reinhold Baer
→ Jutta Hausen →

******Christopher P. Benton, Ph.D******

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These people are all more famous than me.

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Christopher P. Benton, Ph.D**

But my name is in a bigger font!

Also, it was one of von Lindemann's mathematical ancestors, Leonard Euler, who came up with that most beautiful formula that von Lindemann used in his proof. This formula relates the numbers *e*, *i*, π, and *-1.*

$$e^{i\pi} = -1$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Euler

Leopold Kronecker was very critical of von Lindemann's research into pi. Kronecker was also very critical of Cantor's explorations of infinity. Overall, Kronecker is one of the biggest jerks ever in the history of mathematics. In physics, the "jerk" is known as the rate at which acceleration changes over time. Below is the mathematical symbol for the jerk.

$$\frac{d^3 s}{dt^3}$$

Leopold Kronecker
1823 CE – 1891 CE

$$\frac{d^3 s}{dt^3} \rightarrow$$

"What good is your investigation of π? Why study such problems since irrational numbers do not exist?"

And now, here are some more serious estimates of pi from yesteryear to the present.

Serious Pi

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1}{2}}}}\cdots$$

Francois Vieta, 1540 - 1603

Serious Pi

$$\frac{\pi}{2} = \frac{2\times 2\times 4\times 4\times 6\times 6\times 8\times \ldots}{1\times 3\times 3\times 5\times 5\times 7\times 7\times \ldots}$$

John Wallis, 1616 - 1703

Serious Pi

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \ldots$$

4.000000
2.666667
3.466667
2.895238
3.339683
2.976046
3.283738
3.017072

Gottfried Leibniz, 1646 - 1716
James Gregory, 1638 - 1675

Serious Pi

$$\pi = \sqrt{6\left(1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\right)}$$

ultra cool hat →

Leonard Euler, 1707 - 1783

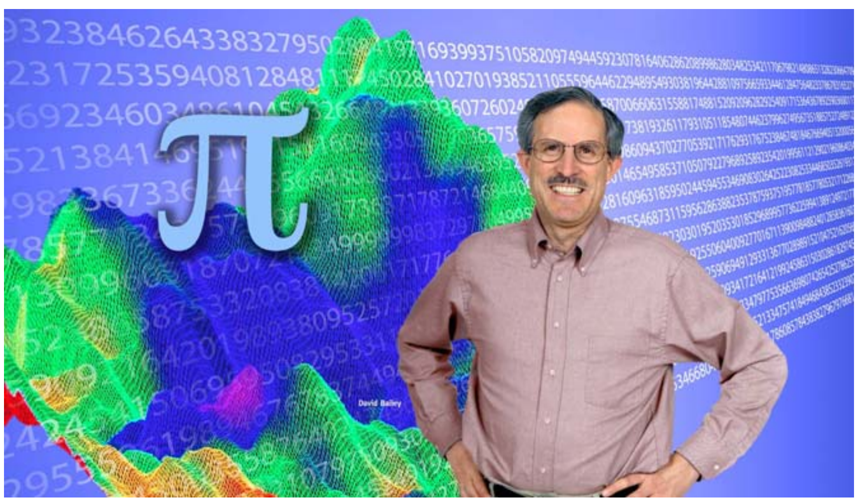
Serious Pi

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \binom{2n}{n}^3 \frac{42n+5}{2^{12n+4}}$$

Ramanujan, 1887 - 1920

Serious Pi

$$\pi = \sum_{n=0}^{\infty}\left(\frac{4}{8n+1}-\frac{2}{8n+4}-\frac{1}{8n+5}-\frac{1}{8n+6}\right)\left(\frac{1}{16}\right)^{n}$$



David Bailey, 1997

3.133333333
3.141422466
3.141587390
3.141592458
3.141592645
3.141592653
3.141592654

In 1989, David and Gregory Chudnovsky built a supercomputer, “m zero,” using mail-order parts, and calculated pi to 1 billion digits.

In 1996, they computed over 8 billion digits.

$$\frac{1}{\pi} = 12\sum_{k=0}^{\infty} \frac{(-1)^k (6k)!(13591409 + 545140134k)}{(3k)!(k!)^3 640320^{3k+3/2}}$$

$$\pi = 3.1415926\ldots5$$

Pi to 1.24 trillion decimal places
Circa 2002
Professor Yasumasa Kanada

I’m going to celebrate “pi day” this year by:

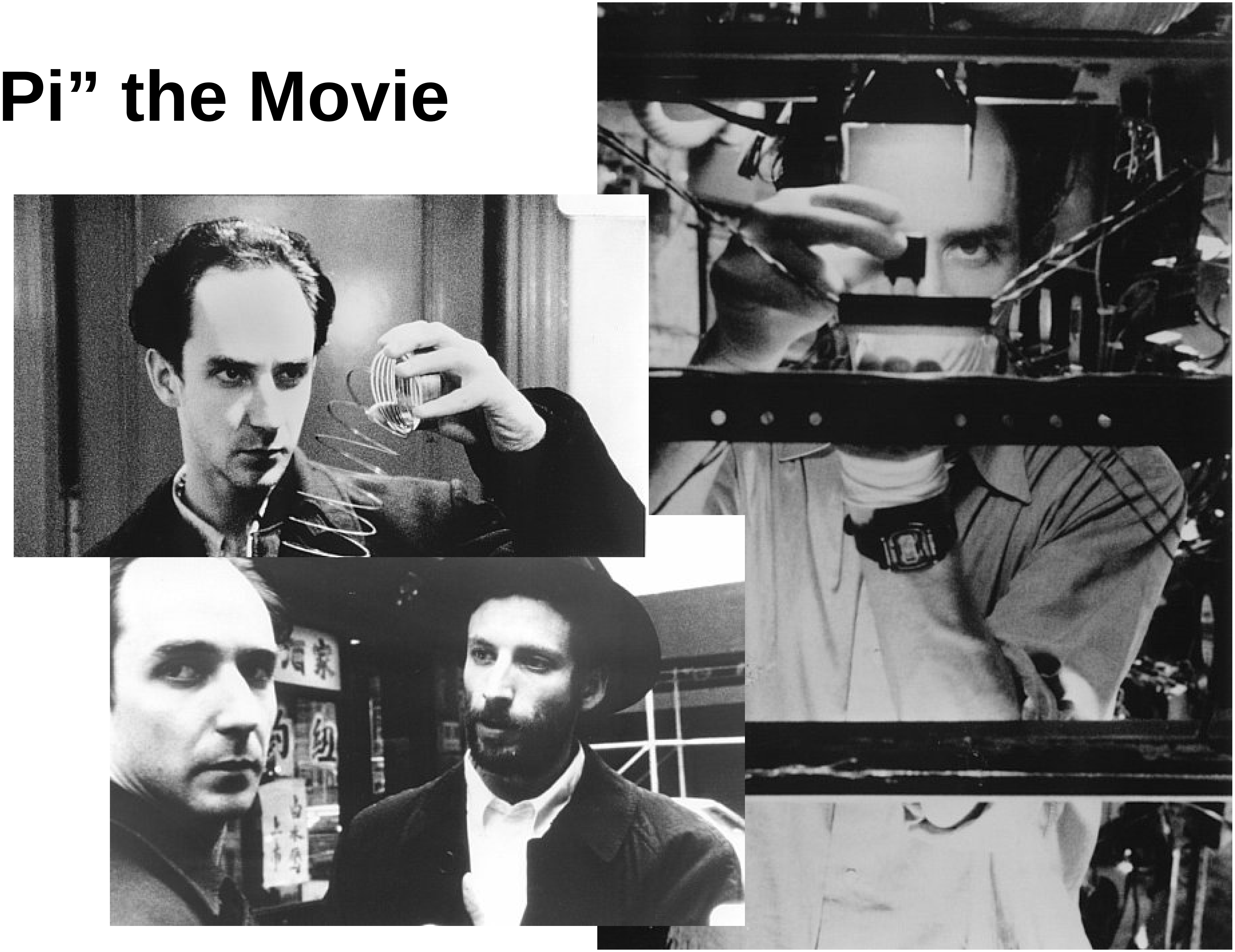
1. Watching “Pi,” the movie.

2. Eating “PIzza.”

3. Rearranging the pieces of my pizza into a rectangle to prove that the area of a circle is πr^2.

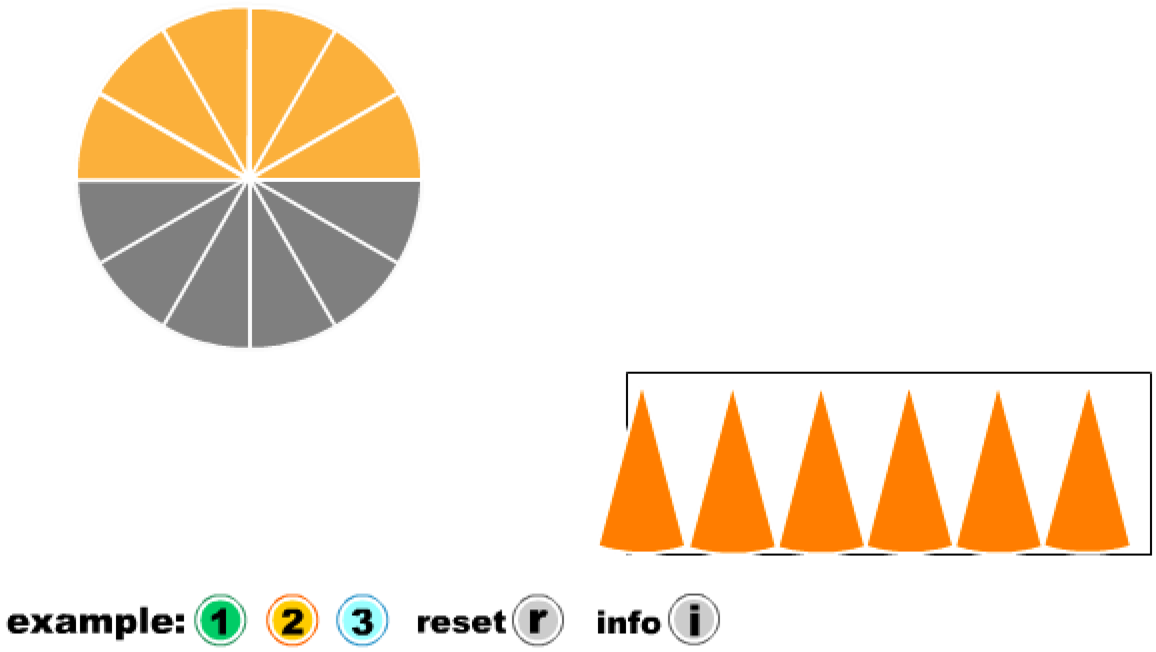
4. Using the Pythagorean Theorem, integration, and parametric equations for a circle to show that the circumference of a circle is $2\pi r$.

“Pi” the Movie



This better be “kosher” bacon and pepperoni!

This picture suggests how to turn a circle into a rectangle in order to find its area.



And now we find the circumference of the circle.

$$\vec{r}(t) = r\cos(t)\hat{i} + r\sin(t)\hat{j}$$

$$0 \le t \le 2\pi$$

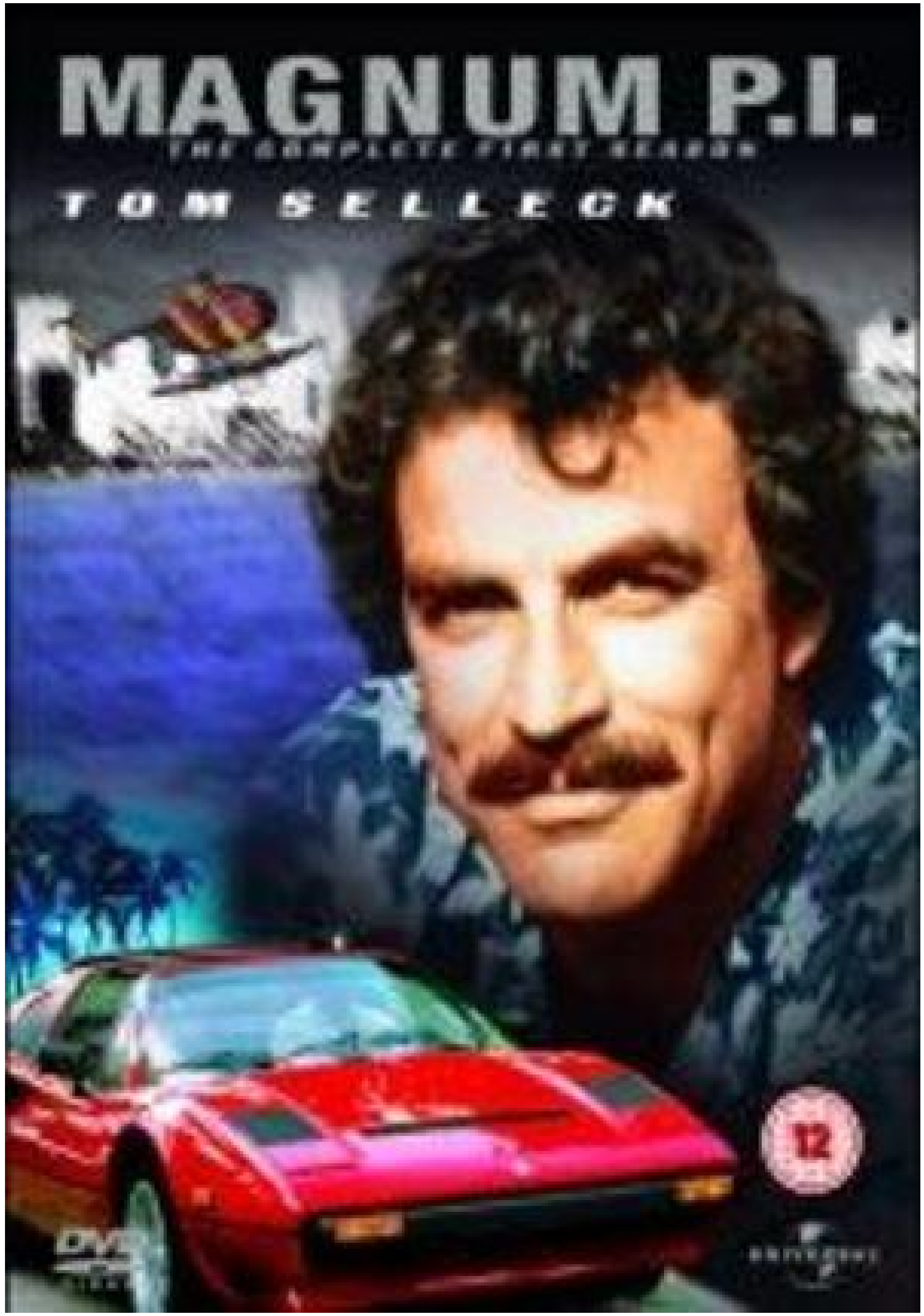
$$r = radius$$

$$\vec{r}'(t) = -r\sin(t)\hat{i} + r\cos(t)\hat{j}$$

$$Arc\,Length = \int_0^{2\pi} \left\|\vec{r}'(t)\right\| dt = \int_0^{2\pi} \sqrt{r^2\sin^2 t + r^2\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{r^2}\, dt = \int_0^{2\pi} r\, dt = rt\Big|_0^{2\pi} = 2\pi r$$

Others will celebrate by watching old episodes of “Magnum PI.”


MAGNUM P.I.
THE COMPLETE FIRST SEASON
TOM SELLECK
12

And, of course, the missus and I will give lots of treats today to “Chloe the Pi Dog.”


Dr. Chloe Continuum, Do.G.
(doctor of growlology)

HAPPY PI DAY!