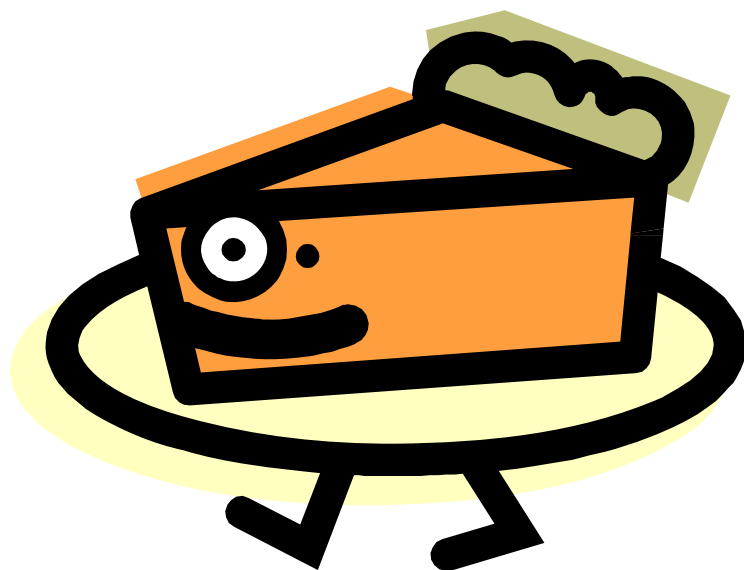


$\pi$  Day



*$\pi$*

$\pi$

Why is this number different from  
all other numbers?

$\pi$

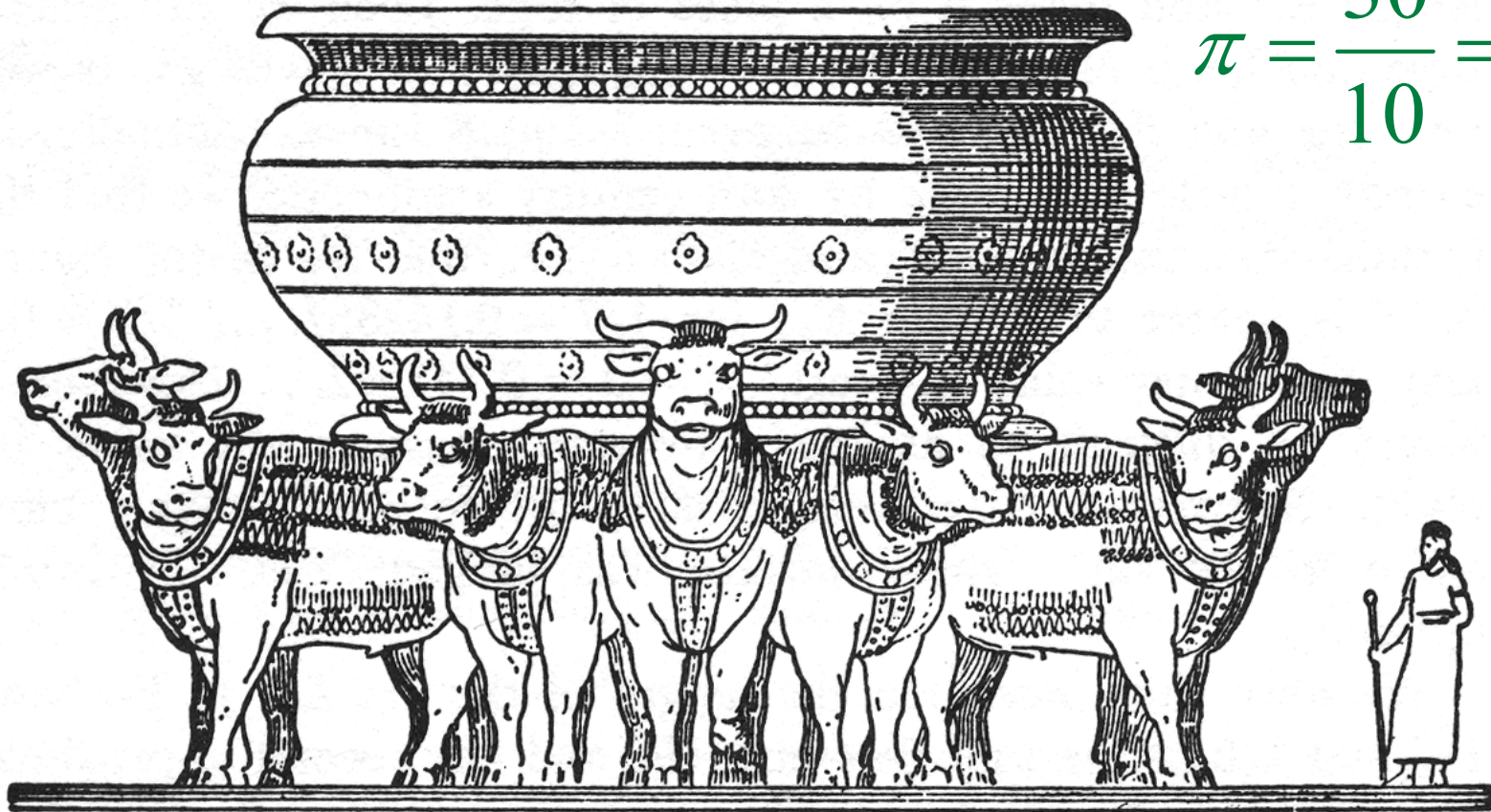
Why is this number different from  
all other numbers?

We begin to explain ...

**A verse in the Bible seems to give an exact value of “3” for pi.**

“He made the sea of a casting ten cubits from one lip to the other lip, circular all around, five cubits its height, a measuring line thirty cubits could encircle it all around.”  
—I Kings 7:23 (circa 550 BCE)

$$\pi = \frac{30}{10} = 3$$



**However, in Hebrew, each letter is also a number, and an odd spelling of the word for “measuring line” seems to indicate how to get a value for pi that is accurate to four decimal places.**

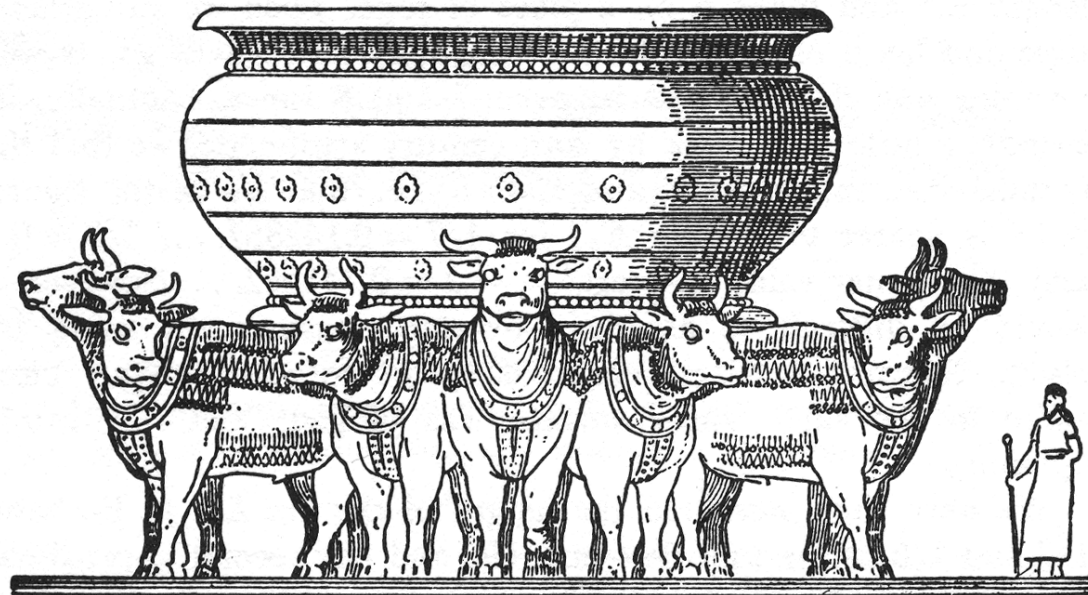
measuring line thirty cubits

קוה שלשים באמה

incorrect → קוה = 111

correct → קו = 106

$$\pi = \frac{30}{10} \cdot \frac{111}{106} = 3.141509434\dots$$





Additionally, there is an extraordinary “Bible code” where the digits 3,1,4,1,5 appear in this verse from First Kings beginning in the middle of the word for “circular.” The length of the skip sequence for this code is 8,976 letters.

ג → 3

ס → 1

ד → 4

ס → 1

ה → 5



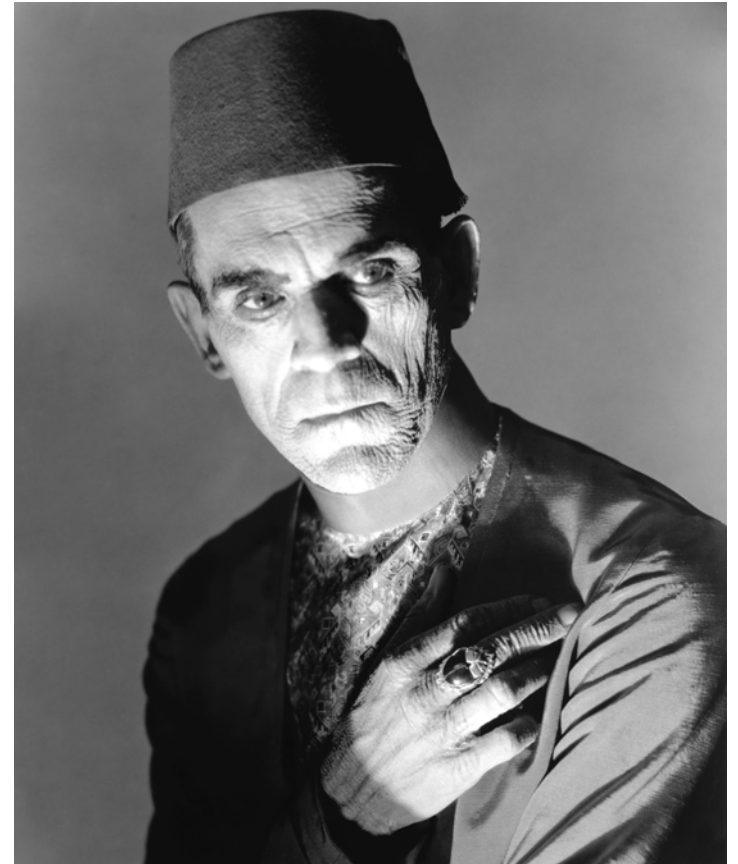
**Here are some early estimates of pi and the dudes that did them.**



# The Rhind Papyrus

**“Cut off 1/9 of a diameter and construct a square of the remainder. This has the same area as the circle.”**

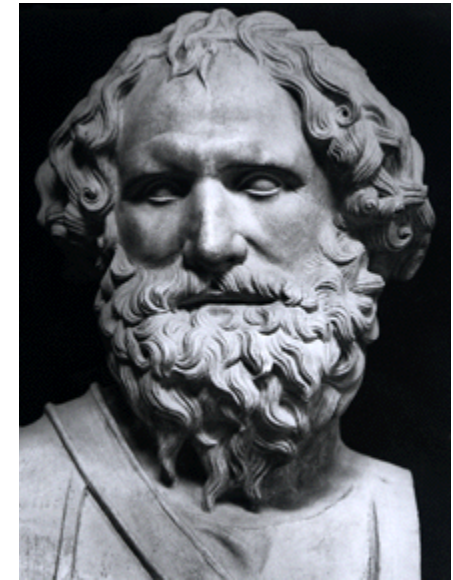
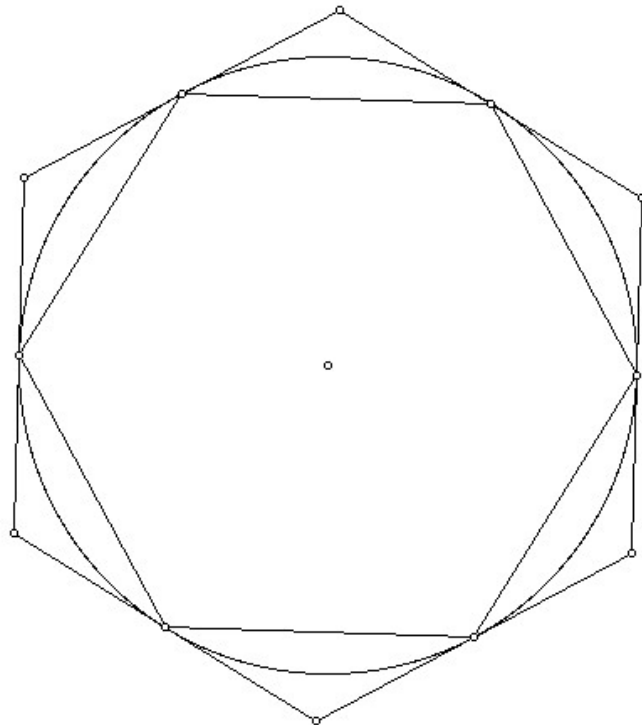
**-Ahmes the scribe  
1650 BCE**



$$\pi = \frac{256}{81} \approx 3.160493827$$

# Archimedes

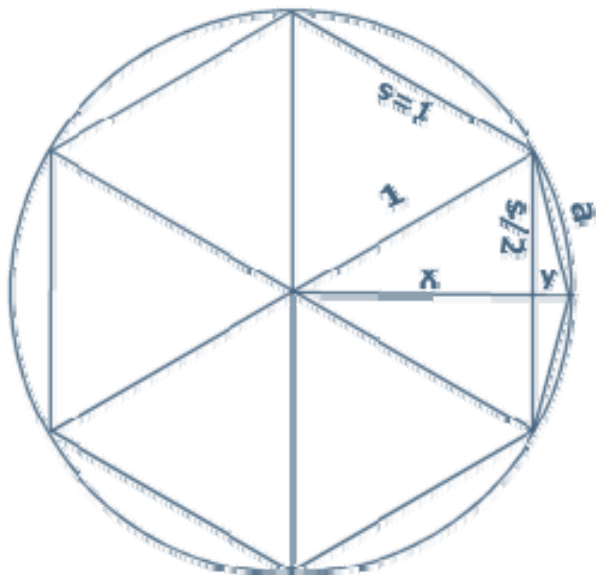
287 BCE – 212 BCE



**“The ratio of the circumference of any circle to its diameter is less than  $3 \frac{1}{7}$ , but greater than  $3 \frac{10}{71}$ .”**

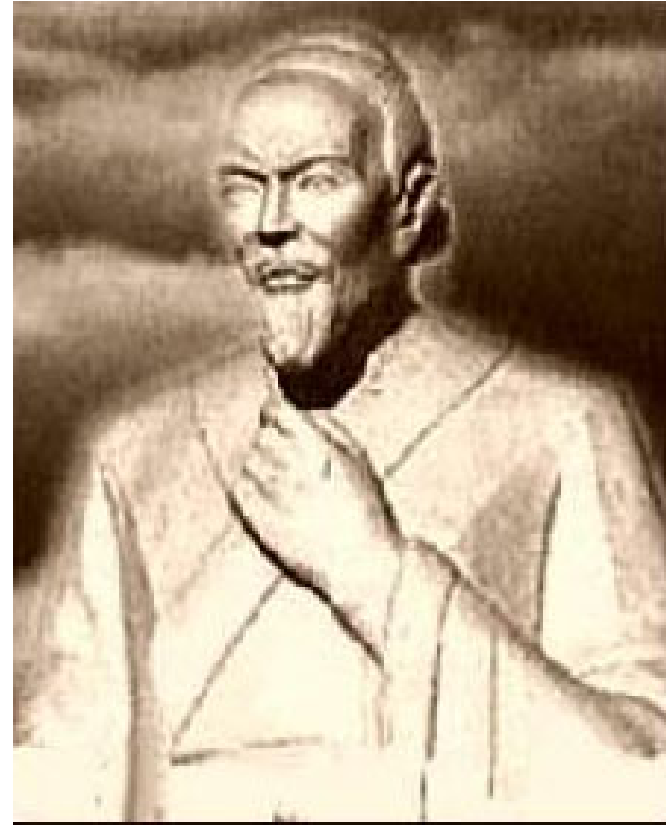
# Claudius Ptolemy

90 – 168 CE



$$\pi = 3^{\circ} 8' 30'' = 3 + \frac{8}{60} + \frac{30}{3600} = 3.141\bar{6}$$

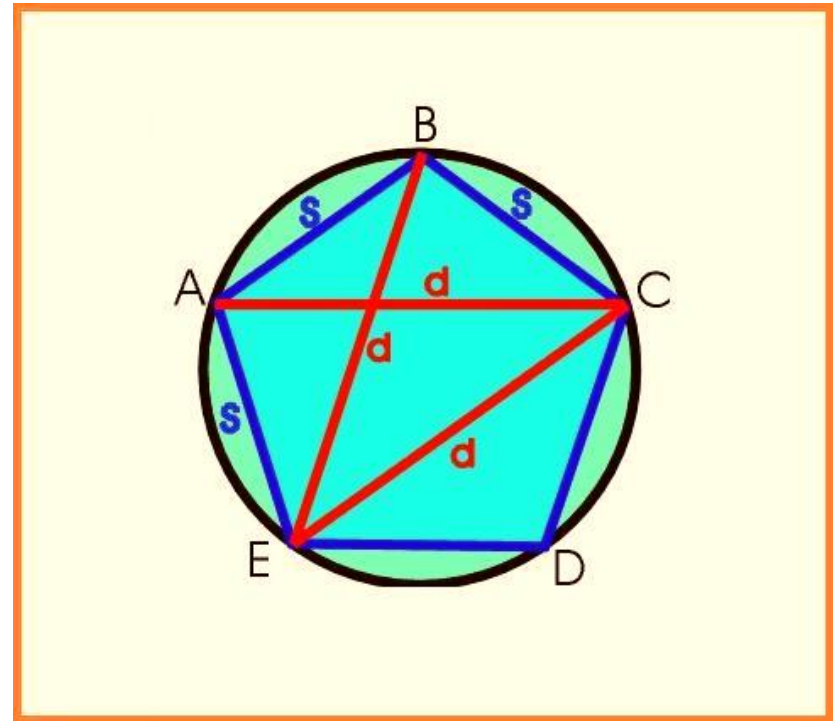
# Tsu Ch'ung Chih b. 429 CE



$$\pi = \frac{355}{113} = 113 \overline{) 355} \begin{array}{r} 3.14159292... \\ \end{array}$$

**This is my favorite! Easy to remember using the odd numbers, 1, 3, 5.**

**Brahmagupta**  
**598 – 668 CE**



$$\pi \approx \sqrt{9.65}, \sqrt{9.81}, \sqrt{9.86}, \sqrt{9.87}, \dots$$

$$\pi \stackrel{?}{=} \sqrt{10} = 3.16\dots \quad (\text{close, but no cigar!})$$



**In 1761, Lambert proved the pi is irrational. I am also irrational.**

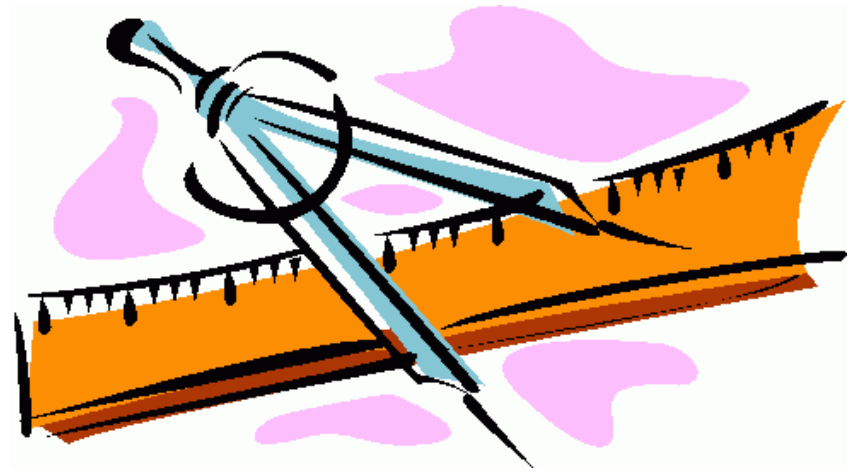
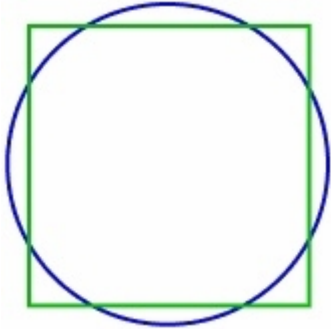
# Johann Heinrich Lambert 1728 – 1777 CE



- ❖ If  $x$  is rational, then  $\tan(x)$  is irrational
- ❖  $\tan(\pi/4) = 1$
- ❖ Therefore,  $\pi/4$  is irrational
- ❖ Therefore,  $\pi$  is irrational (1761)

**Pi is also related to the problem of “squaring the circle.”**

## Squaring the Circle



- ❖ Using a compass and straightedge, construct a square with the same area as a circle
- ❖ Equivalent to constructing a line of length  $\pi$

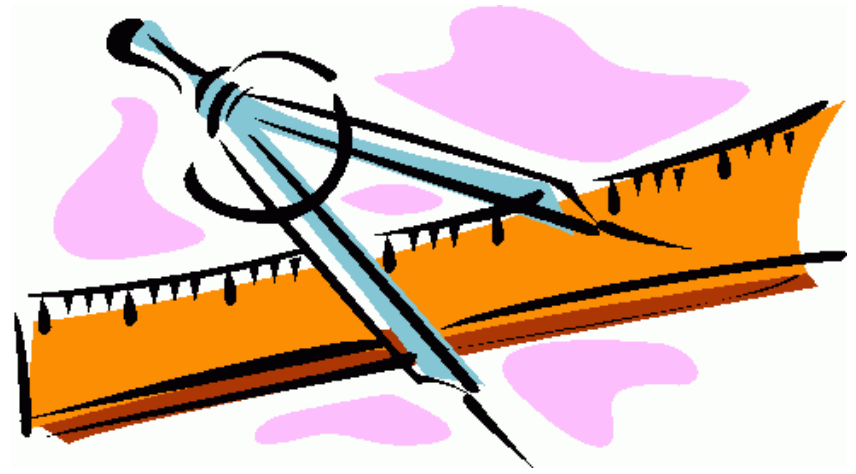
# Euclid

circa 300 BCE

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. **Given any straight line and a point not on it, there "exists one and only one straight line which passes" through that point and never intersects the first line, no matter how far they are extended.**



# Compass & Ruler Constructions



- ❖ Euclid's Postulates →
- ❖ Compass & Ruler Constructions →
- ❖ First and Second Degree Polynomials →
- ❖ Polynomial Equations with Integer Coefficients →
- ❖ Algebraic Numbers →
- ❖ If you can square the circle, then  $\pi$  is algebraic

**Ferdinand von Lindemann proved that pi is  
“transcendental.” I am also transcendental.**

## Ferdinand von Lindemann 1852 – 1939



- ❖ If  $x$  is algebraic, then  $e^x$  is transcendental (i.e. not algebraic)
- ❖ If  $\pi$  is algebraic, then so is  $i\pi$
- ❖  $e^{i\pi} = -1$ , an algebraic number
- ❖ Therefore,  $\pi$  is transcendental (1882)
- ❖ Therefore, you can't square the circle



**Ferdinand von Lindemann's tombstone shows  $\pi$  inside a square.**



⊠  
Universitätsprofessor  
Dr. Ferdinand von Lindemann  
\* 12. April 1862 † 6. März 1939  
Schriftstellerin  
Lisbeth von Lindemann  
geborene Küssner  
\* 22. Juli 1861 † 28. Febr. 1936

**By the way, I'm mathematically descended from Ferdinand von Lindemann. You just go back from me to my advisor in graduate school, then back to her advisor, and so on.**

# THE MATHEMATICS GENEALOGY PROJECT

Gottfried Leibniz → Jacob Bernoulli → Johann Bernoulli →  
Leonhard Euler → Joseph Lagrange  
→ Jean-Baptiste Joseph Fourier, Simeon Poisson  
→ Gustav Dirichlet  
→ Rudolph Lipschitz → Felix Klein  
→ C. L. Ferdinand Lindemann  
→ David Hilbert → Hellmuth Kneser  
→ Reinhold Baer  
→ Jutta Hausen →

**\*\*\*Christopher P. Benton, Ph.D\*\*\***

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**These people are all  
more famous than me.**


**\*\*\*Christopher P. Benton, Ph.D\*\*\***

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→ David Hilbert → Hellmuth Kneser  
→ Reinhold Baer  
→ Jutta Hausen →

**\*\*\*Christopher P. Benton, Ph.D\*\*\***

**But my name is in  
a bigger font!**



**Also, it was one of von Lindemann's mathematical ancestors, Leonard Euler, who came up with that most beautiful formula that von Lindemann used in his proof. This formula relates the numbers  $e$ ,  $i$ ,  $\pi$ , and  $-1$ .**

$$e^{i\pi} = -1$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

Euler



Leopold Kronecker was very critical of von Lindemann's research into pi. Kronecker was also very critical of Cantor's explorations of infinity. Overall, Kronecker is one of the biggest jerks ever in the history of mathematics. In physics, the "jerk" is known as the rate at which acceleration changes over time. Below is the mathematical symbol for the jerk.

$$\frac{d^3 s}{dt^3}$$

**Leopold Kronecker**  
**1823 CE – 1891 CE**

$$\frac{d^3 s}{dt^3} \rightarrow$$

“What good is your investigation of  $\pi$ ? Why study such problems since irrational numbers do not exist?”



**And now, here are some more serious estimates of pi from yesteryear to the present.**

## Serious Pi

$$\frac{2}{\pi} = \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2}}} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{2}}}} \dots$$

Francois Vieta, 1540 - 1603



## Serious Pi

$$\frac{\pi}{2} = \frac{2 \times 2 \times 4 \times 4 \times 6 \times 6 \times 8 \times \dots}{1 \times 3 \times 3 \times 5 \times 5 \times 7 \times 7 \times \dots}$$

John Wallis, 1616 - 1703



## Serious Pi

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \dots$$

**Gottfried Leibniz, 1646 - 1716**  
**James Gregory, 1638 - 1675**

4.000000
2.666667
3.466667
2.895238
3.339683
2.976046
3.283738
3.017072

## Serious Pi

$$\pi = \sqrt{6 \left( 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \right)}$$

ultra cool hat →

Leonard Euler, 1707 - 1783



## Serious Pi

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \binom{2n}{n}^3 \frac{42n + 5}{2^{12n+4}}$$

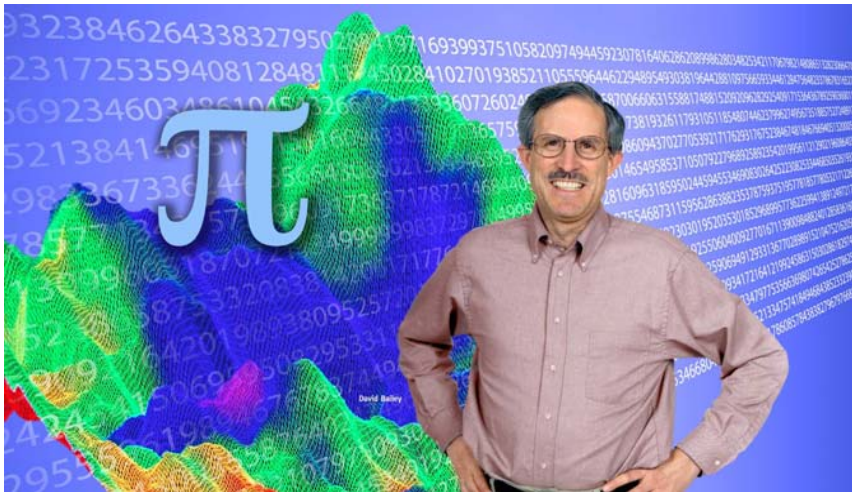
Ramanujan, 1887 - 1920





# Serious Pi

$$\pi = \sum_{n=0}^{\infty} \left( \frac{4}{8n+1} - \frac{2}{8n+4} - \frac{1}{8n+5} - \frac{1}{8n+6} \right) \left( \frac{1}{16} \right)^n$$



David Bailey, 1997

3.133333333
-------------

3.141422466
-------------

3.141587390
-------------

3.141592458
-------------

3.141592645
-------------

3.141592653
-------------

3.141592654
-------------

In 1989, David and Gregory Chudnovsky built a supercomputer, "m zero," using mail-order parts, and calculated pi to 1 billion digits.

In 1996, they computed over 8 billion digits.



$$\frac{1}{\pi} = 12 \sum_{k=0}^{\infty} \frac{(-1)^k (6k)! (13591409 + 545140134k)}{(3k)! (k!)^3 640320^{3k+3/2}}$$

$$\pi = 3.1415926\dots 5$$

300089027768963114810902209724520759167297007850580717186381054967973100  
167870850694207092232908070383263453452038027860990556900134137182368370  
991949516489600755049341267876436746384902063964019766685592335654639138  
363185745698147196210841080961884605456039038455343729141446513474940784  
384423772175154334260306698831768331001133108690421939031080143784334151  
370924353013677631084913516156422698475074303297167469640666531527035325  
467112667522460551199581831963763707617991919203579582007595605302346267  
75794393630746305690108014942714109939136913810725813781357894005599500  
183542511841721360557275221035268037357265279224173736057511278872181908  
449006178013889710770822931002707005005000758909395688148560263224393726  
562472776037890814458837855019107825052704875816470324581290  
378395232453237896029841669225081192186584926770403956481278  
102179913217416305810554598801051121241536374515005635070127  
315926714241342103301566165356028655257222753049998837015348  
7930080626018096238151613669030510919367393835229345888322550  
387064507539473952043968079067086548801682874343786126453815  
3342807530618454859037982179940742536344399602902510015888272  
164745006820704193761584547123039550548239557137256840232268  
213012476794522644820910235647752723082081063518899152692889108455571126  
303965034397896278250016110153235160519655904211844949907789992007329476  
905868577878720982901352956613978884860509786085957017731298155314951681  
467176959760994210036183559138777817698458758104466283998806006162298486  
169353373865787735983361613384133853684211978938900185295691967804554482  
358483701170967212535338758621582310133103877668272115726949518179589754  
393992642197915523385766231676275475703546994148929041301863861194391962

Pi to 1.24 trillion decimal places

Circa 2002

Professor Yasumasa Kanada



**I'm going to celebrate "pi day" this year by:**

- 1. Watching "Pi," the movie.**
- 2. Eating "Pizza."**
- 3. Rearranging the pieces of my pizza into a rectangle to prove that the area of a circle is  $\pi r^2$ .**
- 4. Using the Pythagorean Theorem, integration, and parametric equations for a circle to show that the circumference of a circle is  $2\pi r$ .**

# “Pi” the Movie



**This better be “kosher” bacon and pepperoni!**



This picture suggests how to turn a circle into a rectangle in order to find its area.



example: ① ② ③ reset **r** info **i**

And now we find the circumference of the circle.

$$\vec{r}(t) = r \cos(t)\hat{i} + r \sin(t)\hat{j}$$

$$0 \leq t \leq 2\pi$$

$r = \text{radius}$

$$\vec{r}'(t) = -r \sin(t)\hat{i} + r \cos(t)\hat{j}$$

$$\text{Arc Length} = \int_0^{2\pi} \|\vec{r}'(t)\| dt = \int_0^{2\pi} \sqrt{r^2 \sin^2 t + r^2 \cos^2 t} dt$$

$$= \int_0^{2\pi} \sqrt{r^2} dt = \int_0^{2\pi} r dt = rt \Big|_0^{2\pi} = 2\pi r$$

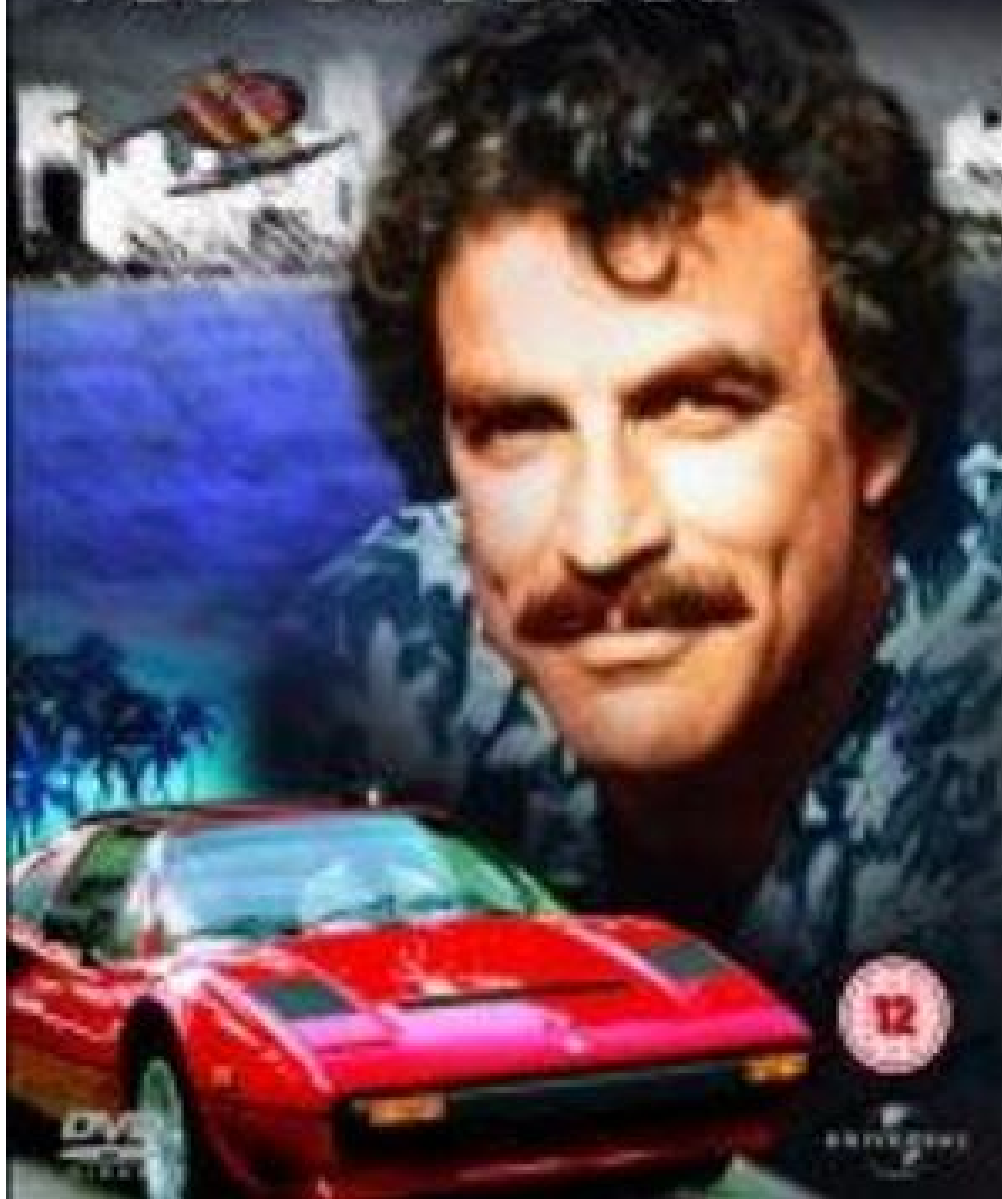


**Others will celebrate by watching old episodes of  
“Magnum PI.”**

# MAGNUM P.I.

THE COMPLETE FIRST SEASON

TOM SELLECK



**And, of course, the missus and I will give lots of treats today to “Chloe the Pi Dog.”**

# Dr. Chloe Continuum, Do.G.

(doctor of growlology)



HAPPY PI DAY!

