Einstein's Theory of Special Relativity Made Relatively Simple!

by
Christopher P. Benton, PhD

Young Einstein

Albert Einstein was born in 1879 and died in 1955. He didn't start talking until he was three, and at age nine he still didn't talk very well. Everyone thought he was retarded. However, he got smarter.

The Ether

In 1820, Thomas Young performed an experiment that indicated that light is composed of waves. However, common sense told the physicists of that day that every wave needs some sort of medium to wave through. For example, ocean waves wave through water, and sound waves wave through air (as well as water and other materials). Thus, physicists believed that light waves also needed some medium to wave through. They called this medium the **ether**.

In 1831 two American scientists, Michelson and Morley, set up an experiment to detect the ether. The idea behind their experiment was that as the earth moves through space it would at times be moving with the ether and at other times against the ether. Suppose the earth were moving against the ether. Then the situation would be analogous to moving upstream in a boat. In that case, anything you threw downstream would move away from you faster than something that you threw upstream. Thus, physicists reasoned that as the earth moved through the ether, the speed at which light moved in one direction would be different from the speed of light in another direction. The experiment of Michelson and Morley was designed to detect this difference in speed, and thus, confirm the existence of the ether. However, when performed, the Michelson-Morley experiment detected no variation in the speed of light. As a result, scientists gradually discarded the idea of the ether (since it couldn't be detected), and they began to accept the idea that the speed of light is the same in all directions.

Thus, Einstein began his theory of special relativity with two assumptions:

1. <u>The Principle of Relativity</u>: One cannot tell by any experiment whether one is at rest or moving uniformly (that is, moving in a straight line with constant velocity). In other words, **there is no such thing as absolute rest**. All motion or rest is only in relation to other observed objects (i.e. I can consider myself not to be moving with respect to the earth while at the same time I am moving very rapidly with respect to the sun).

2. <u>The Constancy of the Speed of Light</u>: The speed of light in a vacuum has the same value **c** with respect to any observer either at rest or moving uniformly (c = 186,282.397 miles per second).

What Einstein then discovered was that in order for all observers to measure the same velocity for a beam of light, time would have to "flow" differently for different observers, and space would sometimes have to contract.

Experiment 1: Mrs. Einstein is standing in a field and Mr. Einstein is riding on a railroad car that is moving with velocity v. Mr. Einstein shines a flashlight in the direction in which he is moving.
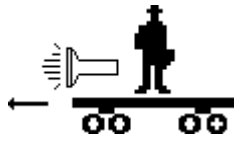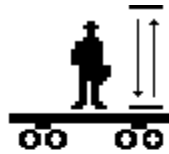

Question: What happens?

Answer: Because of the principle of the constancy of the velocity of light, each observer will measure the light beam from the flashlight as traveling at the same speed. This may be contrary to what you expected as you might have thought that the observer in the field would have seen the beam moving at (the speed of light) + (the speed of the train). Nevertheless, this is not what is observed in practice. What actually occurs in the real world is that no one ever measures light moving faster or slower than c ≈ 186,000 miles per second (in a vacuum).

Experiment 2: Mrs. Einstein is standing in a field. Next to her is a light clock. That is, two mirrors that are reflecting a beam of light back and forth, and the journey from one mirror to the other and back again counts as one tick of the clock. Also, Mrs. Einstein is wearing a watch that is synchronized with her light clock.

Standing on a railroad car is Mr. Einstein. He also has a light clock, and his clock is synchronized with Mrs. Einstein's and his own wristwatch. The railroad car is not moving.

Question: What happens?

Answer: Nothing unusual happens. Mr. Einstein's watch and clock stay perfectly synchronized with Mrs. Einstein's.

Experiment 3: We now have the same set up except that the railroad car is now moving to the left with a velocity *v*.
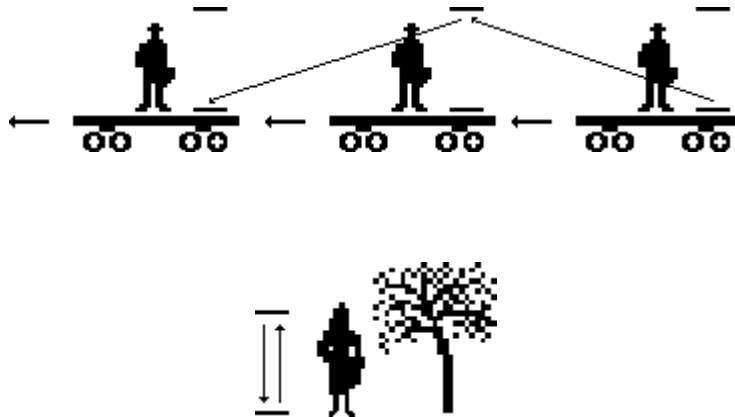
Question: What happens?

Answer: From Mr. Einstein's perspective, the beam of light keeps going up and down between the mirrors, but from Mrs. Einstein's perspective, the light now has to travel a diagonal path from one mirror to the other. Since Mrs. Einstein still measures the speed of light as c, she is now going to observe Mr. Einstein's light clock as ticking slower than hers since the light now has a longer distance to travel. However, since Mr. Einstein still experiences his watch as being synchronized with his clock, Mrs. Einstein will see his watch slow down along with his clock!

Conclusion: If someone moves in a straight line with velocity *v* with respect to you, then you will observe time passing more slowly for them.

With a little algebra we can compute exactly how much time will slow down. Let,

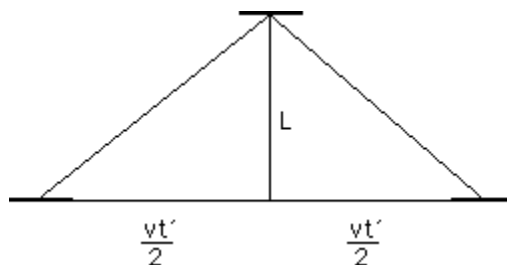L= length or height of the light clocks
v = Mr. Einstein's velocity
t = time between ticks on Mrs. Einstein's clock
t' = time between ticks on Mr. Einstein's clock <u>as Mrs. Einstein observes it</u>
c = speed of light.

We will make frequent use of the formula distance = rate **x** time. We now compute the distance that the light travels in two ways.

On the one hand, the distance traveled is ct´ (distance = rate **x** time). But on the other hand, using the Pythagorean Theorem, we have that the distance is

$$2\sqrt{L^2 + \left(\frac{vt'}{2}\right)^2} = 2\sqrt{L^2 + \frac{v^2t'^2}{4}} = 2\sqrt{\frac{4L^2 + v^2t'^2}{4}} = \frac{2\sqrt{4L^2 + v^2t'^2}}{\sqrt{4}} = \sqrt{4L^2 + v^2t'^2}.$$

Hence, $ct' = \sqrt{4L^2 + v^2t'^2}$

which implies that $c^2t'^2 = 4L^2 + v^2t'^2$,

which implies $c^2t'^2 - v^2t'^2 = 4L^2$,

which implies $t'^2(c^2 - v^2) = 4L^2$,

which implies $t'^2 = \dfrac{4L^2}{c^2 - v^2} = \dfrac{4L^2}{c^2\left(1 - \dfrac{v^2}{c^2}\right)}$

which implies $t' = \sqrt{\dfrac{4L^2}{c^2\left(1 - \dfrac{v^2}{c^2}\right)}} = \dfrac{\sqrt{4L^2}}{\sqrt{c^2}\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{2L}{c\sqrt{1 - \dfrac{v^2}{c^2}}}.$

Since t = (2L)/c (i.e. time = distance/rate), this gives us

$$t' = \frac{t}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

 Therefore, if t is the time between ticks on Mrs. Einstein's watch, then she will observe a longer interval of

$$t' = \frac{t}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

between ticks on Mr. Einstein's watch! Notice that this difference is not very much unless one is traveling at an extremely fast velocity.

Experiment 4: Super Einstein is flying through space with his twin brother, Murray, who is 186,000 miles behind him. Every time he wants to make an acceleration of 10 mph, he uses a flashlight to signal his brother so that they will accelerate together and stay the same distance apart.

From Einstein's perspective, he and his brother are 186,000 miles apart, and by letting 1 second pass on his clock before accelerating, he allows the light to reach his brother right at the moment of acceleration. As a result, from Einstein's point of view he and his brother accelerate together and remain a constant 186,000 miles apart. This is what Einstein sees, but if we are standing still with respect to Einstein and his brother, then what will we see?

Answer: From our perspective, two things happen. First, we say that the beam of light has less than 186,000 miles to travel since his brother is traveling toward it. Second, we are going to see Einstein's clock as running slow. Thus, for two reasons we are going to see Einstein's brother get the signal to accelerate before a full second has passed on Einstein's clock and he begins his own acceleration. Hence, the distance between Einstein and his brother gets shorter. However, if we stop our analysis at this point, then we are going to wind up with a contradiction, because if Einstein and his brother keep accelerating, and if we keep seeing his brother accelerate first, then eventually distance between them will become so small that Murray will run into Einstein. However, from Einstein's perspective, he and his brother stay a constant 186,000 miles apart! How can we resolve this seeming contradiction? Only by making a very bizarre assumption. In order to keep Murray from running into his brother <u>the shortening of the distance between Einstein and his brother must be compensated for by a contraction of length in a direction parallel to the direction in which Einstein and his brother are moving</u>! In other words, from our point of view, Einstein and his brother are getting shorter so that some distance always remains between them in spite of their accelerations.

In the next experiment we will derive a formula that will show us by just how much lengths contract.

Experiment 5: Next to Einstein, a light clock lies on its side on a railroad car as the car moves to the left with velocity *v*.
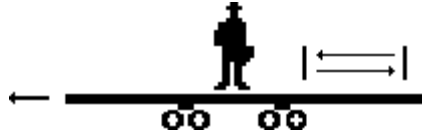
Question: How is the length of the clock affected?

Answer: We know that the time between ticks on the moving clock, as we see it, will be

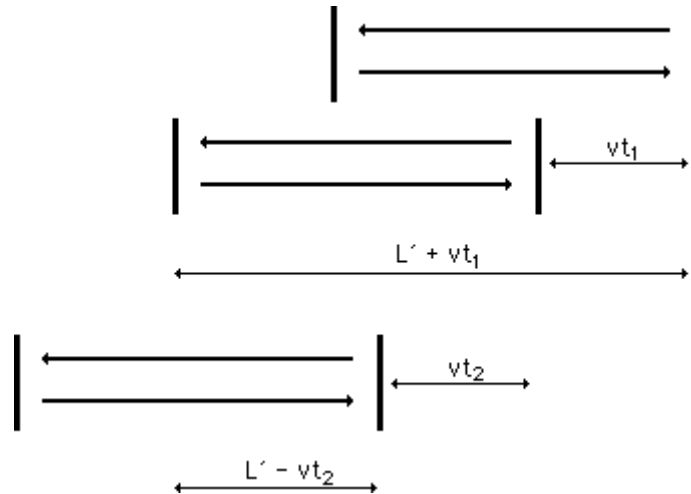$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Let,

t_1 = time it takes the light to go from the first mirror to the second (right to left), as we perceive it.
t_2 = time that it takes the light to go from the second mirror back to the first, as we perceive it.
L' = length of the light clock, as we perceive it.

We can analyze the situation as follows:

From the above picture we see that the distance the light traveled in time t_1 was $L' + vt_1$. However, this distance is also equal to ct_1 (rate **x** time). Also, the distance the light traveled on the return trip was $L' - vt_2 = ct_2$. Solving for t_1, we have

$$L' + vt_1 = ct_1$$

which implies that $L' = ct_1 - vt_1$,

which implies $L' = (c - v)t_1$,

which implies $t_1 = \dfrac{L'}{c - v}$.

Similarly,

$$L' - vt_2 = ct_2$$

which implies that $L' = ct_2 + vt_2$,

which implies $L' = (c + v)t_2$,

which implies $t_2 = \dfrac{L'}{c + v}$.

Thus,

$$t' = t_1 + t_2$$

$$= \dfrac{L'}{c - v} + \dfrac{L'}{c + v}$$

$$= \dfrac{L'c + L'v + L'c - L'v}{c^2 - v^2}$$

$$= \dfrac{2L'c}{c^2 - v^2}$$

which implies that $\dfrac{t}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{2L'c}{c^2 - v^2}$.

Since $t = (2L)/c$,

$$\dfrac{2L}{c\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{2L'c}{c^2 - v^2}$$

which implies that $\dfrac{2L}{c\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{2L'c}{c^2\left(1 - \dfrac{v^2}{c^2}\right)}$,

which implies $\dfrac{2L}{c\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{2L'}{c\left(1 - \dfrac{v^2}{c^2}\right)}$,

which implies $\dfrac{L}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{L'}{1 - \dfrac{v^2}{c^2}}$,

which implies $L' = L\sqrt{1 - \dfrac{v^2}{c^2}}$.

Conclusion: Since $\sqrt{1 - \dfrac{v^2}{c^2}}$ is less than 1, this shows that we will measure the length of Einstein's light clock as being less than ours. Thus, when an object is moving in a straight line with a fixed velocity v, we will see its length, as measured in the direction in which it is moving, shorten.

Experiment 6: Einstein's wife wants him to jump through hoops, but he wonders if anything strange will happen with regard to lengths that are perpendicular to his direction of motion.
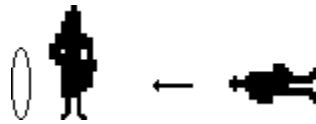
Conclusion: Let us suppose that when an object moves that there is a contraction of length in the direction that is perpendicular to its line of motion. Then the diagrams below will show that this assumption leads to a contradiction.

By this assumption, if the hoop considers itself as standing still, then Einstein shrinks in the direction perpendicular to his motion and has no trouble going through the hoop.
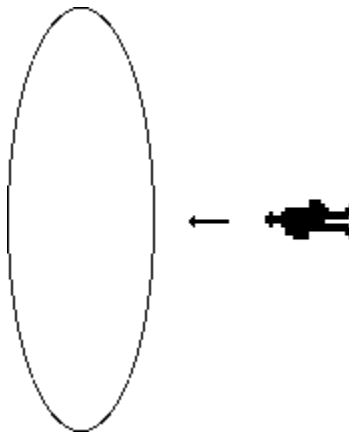
On the other hand, if Einstein considers himself as standing still and the hoop as moving toward him, then by our assumption he would see the hoop contract, and conceivably, he might then be unable to pass through the hoop.

This is a contradiction, though, since we can't have Einstein both passing through and not passing through the hoop. The assumption that leads to this contradiction is that lengths perpendicular to our line of motion will contract. An assumption that these lengths would expand leads to a similar contradiction. Thus, we can only conclude that there will be no change at all in lengths that are perpendicular to our line of motion. The only lengths that change are those that are parallel to our line of motion, as we showed in the previous experiment.

Experiment 7: Mrs. Einstein is standing in a field and she sees two simultaneous flashes of lightning. Mr. Einstein is on a railroad car moving to the left with velocity v.
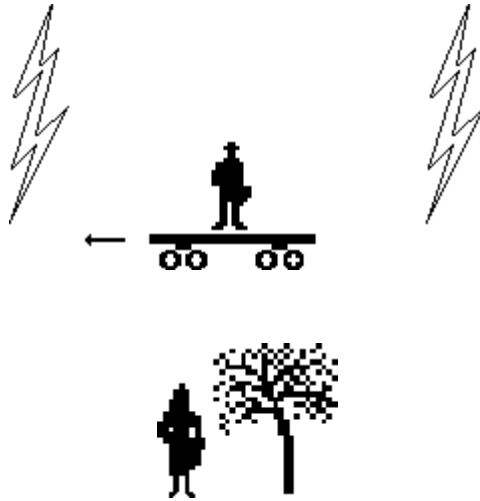
Question: What will Mr. Einstein see?

Answer: Mr. Einstein will see the light flash on the left first since he is moving toward that light and away from the flash on the right. Thus, he will not agree that the flashes of light occurred at the same time. In fact, from his perspective, Mrs. Einstein is the one that is moving, and he will say that it is only because of her movement that she happened to perceive both flashes of light as happening simultaneously.

Experiment 8: Mr. Einstein is on a railroad car moving to the left with velocity v, and on his car are two light bulbs that, from his perspective, come on simultaneously. To confirm this, he could also rig some sort of detector that would go off only if both beams of light arrive at his position simultaneously.
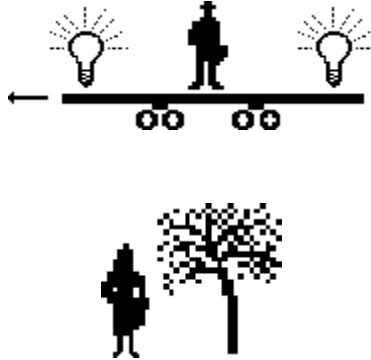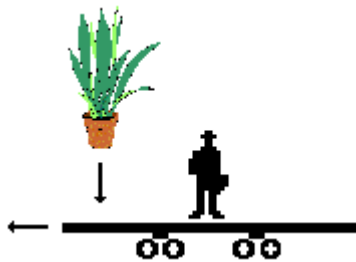

Question: What will Mrs. Einstein see?

Answer: She will agree that both beams of light reach Mr. Einstein at the same time. However, since from her point of view the light on the right has greater distance to travel, she will see the light on the right come on first!

Conclusion: From the above experiments we see that events which may be simultaneous for one observer can happen in a different sequence for another observer. This leads us to the startling conclusion that there is no such thing as a universal "now" for which everyone will agree on what happens "now". That is, I can see two events as happening "now" while another observer will see one event happening "now" with the other event yet to occur!

Experiment 9: Mr. Einstein drops his wife's favorite potted plant while traveling at a high velocity. It shatters. Meanwhile, his wife is standing in the field watching.

Question: What will they each observe?

Answer: They both agree that the pot shatters when it hits the surface and that Mr. Einstein is in BIG trouble. However, from Mrs. Einstein's point of view, Mr. Einstein's time slows down and she sees the plant fall in slow motion. This raises the question of why the pot shatters if it seems to gently float to the surface of Mr. Einstein's railroad car. The answer has to do with momentum. Momentum is the amount of "punch" something has. Physicists define momentum as the product of mass and velocity (momentum = mass **x** velocity). For instance, a car hitting you at 1 mph will do a lot more damage than a feather hitting you at 1 mph because it has more "punch". This extra punch or momentum is a consequence of the car's greater mass. Likewise, if both Mr. and Mrs. Einstein agree that the dropped pot shatters, then this is because the momentum of the pot has remained unchanged. However, since Mrs. Einstein sees the pot falling with less velocity, this also means that **its mass must have increased** in order for momentum to be conserved.

Conclusion: When an object goes fast with respect to us, not only do we see its clocks slow down and its length shorten, we also see an increase in its mass!

Conclusions

An Obvious Objection

Objection: He says my clock slows down and that my distances shrink when I know that it is really his time and space that are changing.

Answer: Actually, there is no contradiction to each of us observing the other's clocks and distances changing. There would only be a contradiction if both of us were making our measurements from the same frame of reference and then arrived at different results. However, in order to be in the same frame of reference, one of us would have to alter his motion in order to bring it in line with the other person's. When this happens, extra forces are brought into play and that person is no longer moving with a constant velocity with respect to us. This destroys the symmetry of the situation, and results in one of us definitely having a slower clock.

Experimental Verification: Several relativistic effects have been confirmed experimentally. For example, highly accurate atomic clocks have been synchronized with one another and then one has been flown at high speeds in a jet for a sufficient length of time, and when compared with each other again, less time is found to have passed on the clock that was moving at high speeds. It has also been observed that atomic particles decay more slowly when moving at high speeds as predicted by relativity. In addition, many other experiments have been performed over the years that provide ample verification of the predictions of relativity.

Final Conclusions: Space and time are stranger than we imagine, and our usual ideas of space and time are just as inaccurate as the belief that people once had that the earth is flat. Much of our confusion results from the assumption that time and space are things that exist independently of objects. However, try to imagine a universe in which no objects exist. In such a universe there would be no points of reference from which to measure the passage of time or distances in space. In such an existence, time and space would only exist as mental constructs. Thus, if we can accept that time is only a measurement that is made of the separation between parts of an <u>observed</u> sequence of events and that distance is only a measurement of the separation between events that are <u>observed</u> to occur at the same time, then it is not so surprising that observers in different frames of reference will measure these things differently. In other words, the universe is relatively . . . interesting!