OPTIMIZATION

DEFINITION:

Let *p* be a point in the domain in *f*.

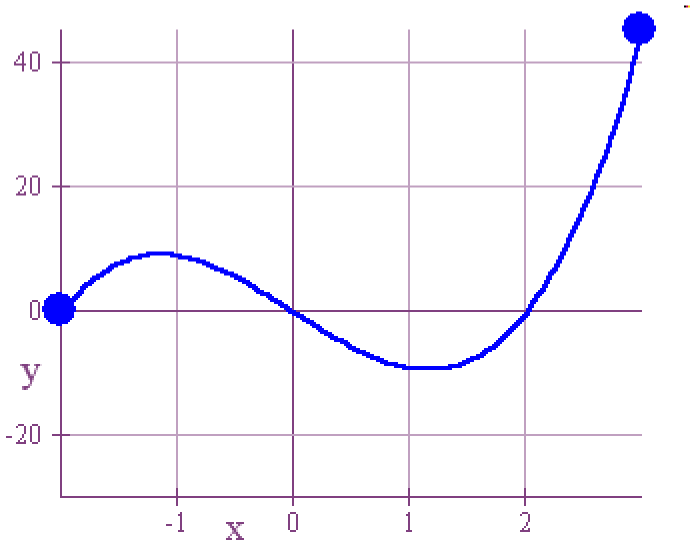
- The function *f* has a ***global minimum*** at *p* if *f*(*p*) is less than or equal to all values of *f*.
- The function *f* has a ***global maximum*** at *p* if *f*(*p*) is greater than or equal to the values of *f*.

THE EXTREME VALUE THEOREM:

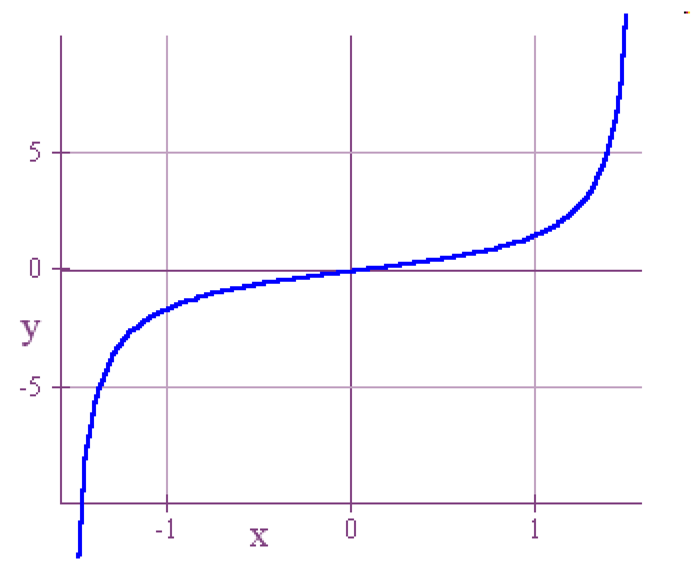
If $f(x)$ is continuous on the closed interval [a,b], then $f(x)$ has a *global maximum* and a *global minimum* on that interval.

QUESTION:

Why does the theorem apply only to a closed interval?



$$f(x) = x^3 - 12x$$
$$-2 \le x \le 3$$



$$f(x) = \tan x$$
$$-\frac{\pi}{2} < x < \frac{\pi}{2}$$

Strategy for Finding Global Extrema on a Closed Interval:

- Find the function values of the critical points in the open interval.

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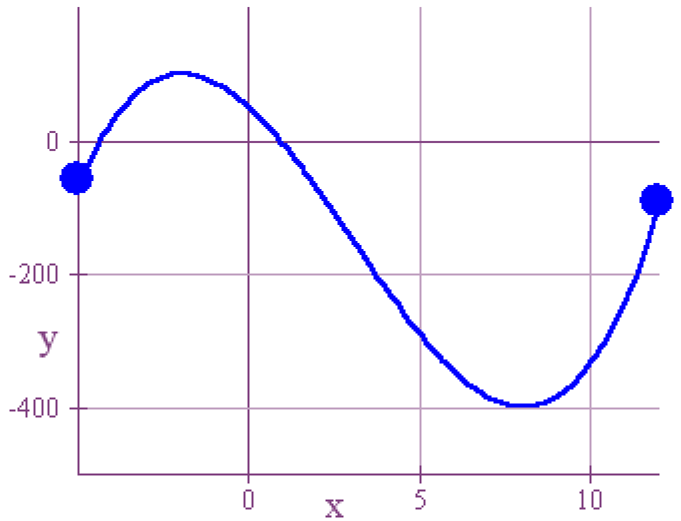
- Find the function values of the critical points in the open interval.
- Find the function values of the endpoints of the interval.
- Identify your global maximum and minimum.

Example:

$$f(x) = x^3 - 9x^2 - 48x + 52 \quad \text{on} \quad [-5,12]$$



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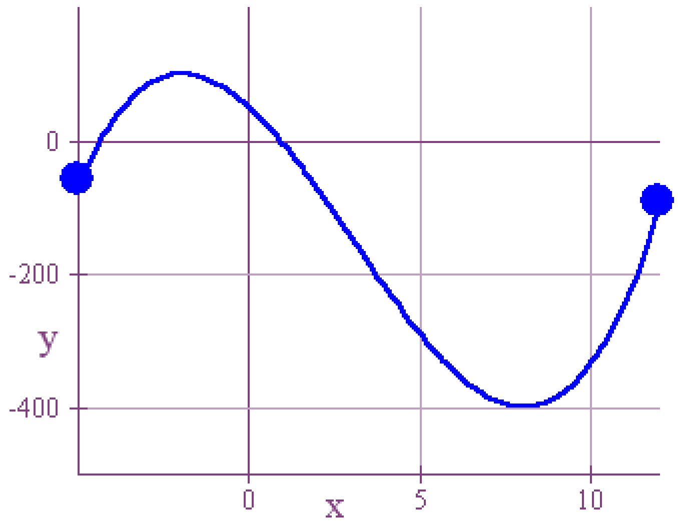

$$f'(x) = 3x^2 - 18x - 48$$

$$= 3(x+2)(x-8)$$

Example:

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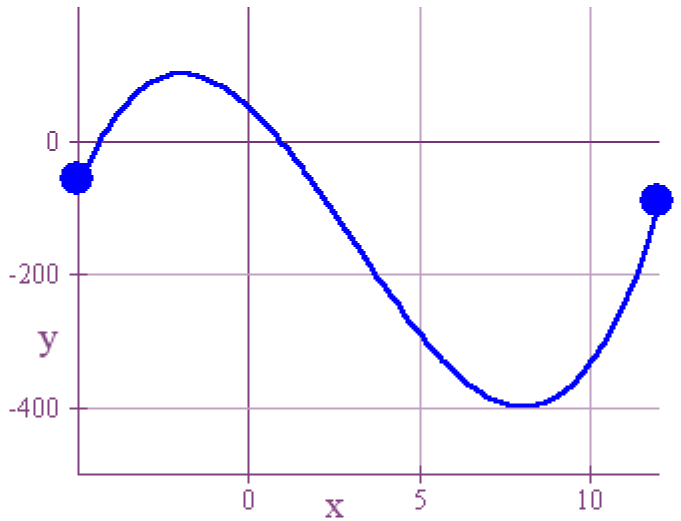


$$f(-5) = -58$$

$$f(-2) = 104 \leftarrow \text{ global max}$$

$$f(8) = -396 \leftarrow \text{ global min}$$

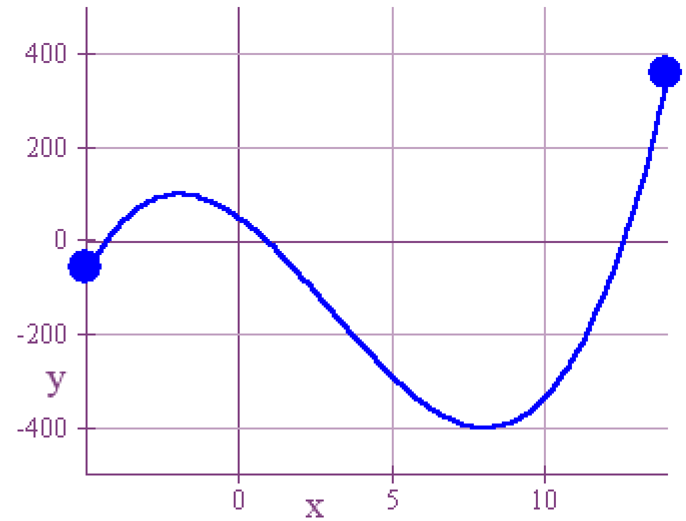
$$f(12) = -92$$

Example:

$$f(x) = x^3 - 9x^2 - 48x + 52 \quad \text{on} \quad [-5,14]$$

$$f'(x) = 3x^2 - 18x - 48$$

$$= 3(x+2)(x-8)$$



$$f(-5) = -58$$

$$f(-2) = 104$$

$$f(8) = -396 \leftarrow \text{ global min}$$

$$f(14) = 360 \leftarrow \text{ global max}$$

Example:

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$$f'(x) = 3x^2 - 18x - 48$$

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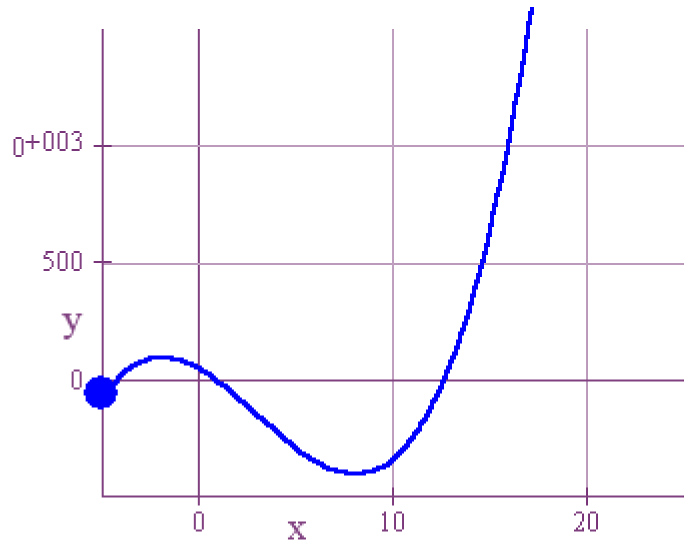


$$f(-5) = -58$$

$$f(-2) = 104$$

$$f(8) = -396 \leftarrow \text{ global min}$$

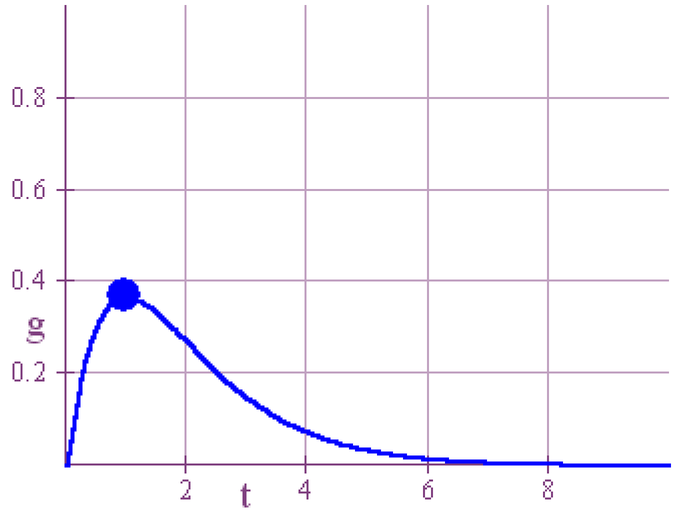
no global max

Section 4.2, Problem 18:

$$g(t) = te^{-t} = \frac{t}{e^t} \quad \text{on} \quad (0, \infty)$$

$$g'(t) = -te^{-t} + e^{-t}$$

$$= e^{-t}(-t+1)$$



$$g(1) = \frac{1}{e} \approx 0.3678794412 \leftarrow$$ global max

no global min

Section 4.2, Problem 26:

For some positive constant *C*, a patient's temperature change, *T*, due to a dose, *D*, of a drug is given by:

$$T = \left(\frac{C}{2} - \frac{D}{3} \right) D^2 = \frac{C}{2} D^2 - \frac{1}{3} D^3$$

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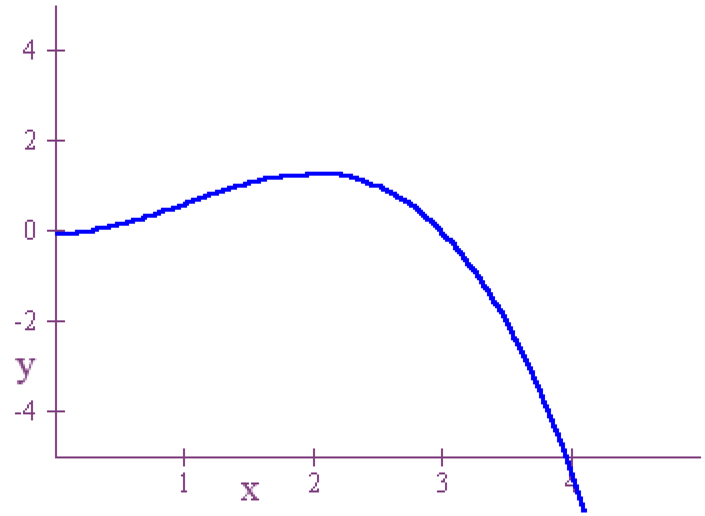
What dosage maximizes the temperature change?

$$T = \frac{C}{2}D^2 - \frac{1}{3}D^3, \qquad C > 0, D \geq 0$$

HINT: Think, too, about the long-term behavior of this function.

$$T'(D) = CD - D^2 = D(C - D)$$



sample graph

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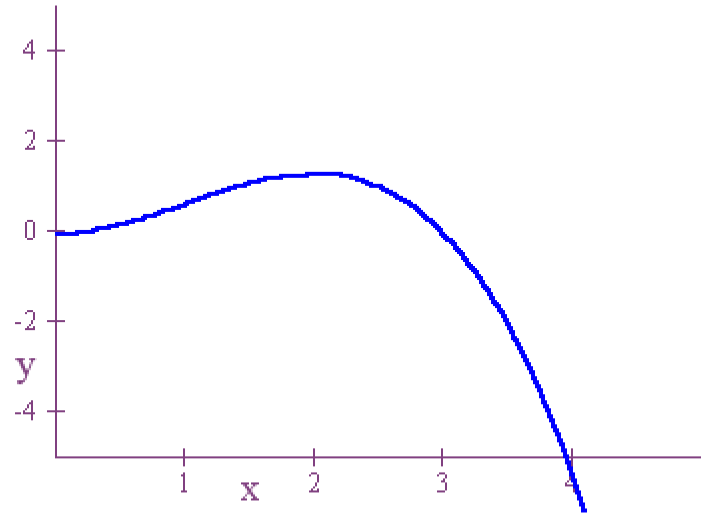

sample graph

Maximum temperature change when $D = C$.

For some positive constant *C*, a patient's temperature change, *T*, due to a dose, *D*, of a drug is given by:

$$T = \left(\frac{C}{2} - \frac{D}{3}\right)D^2 = \frac{C}{2}D^2 - \frac{1}{3}D^3$$

$$T'(D) = CD - D^2$$

The sensitivity of the drug is defined as dT/dD. What dosage maximizes sensitivity?

For some positive constant *C*, a patient's temperature change, *T*, due to a dose, *D*, of a drug is given by:

$$T = \left(\frac{C}{2} - \frac{D}{3}\right)D^2 = \frac{C}{2}D^2 - \frac{1}{3}D^3$$

$$T'(D) = CD - D^2$$

$$T''(D) = C - 2D$$

$$T''(D) = 0 \ \text{ if } D = \frac{C}{2}$$

$$T''(D) > 0 \ \text{ if } D < \frac{C}{2}$$

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The sensitivity is maximized when D=C/2.

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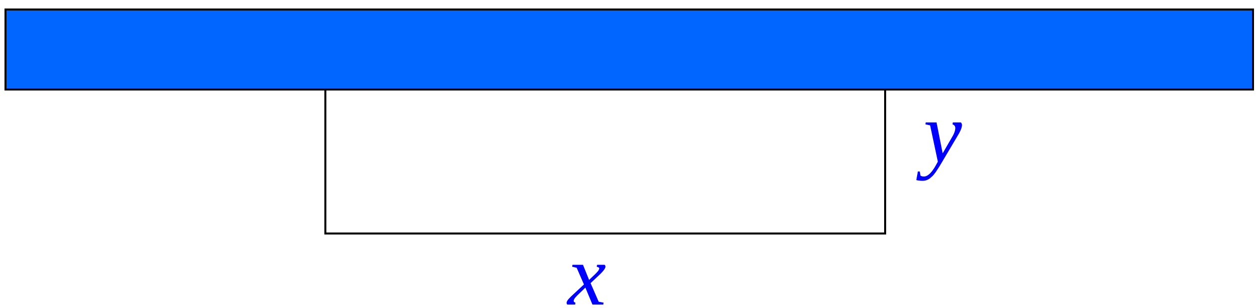
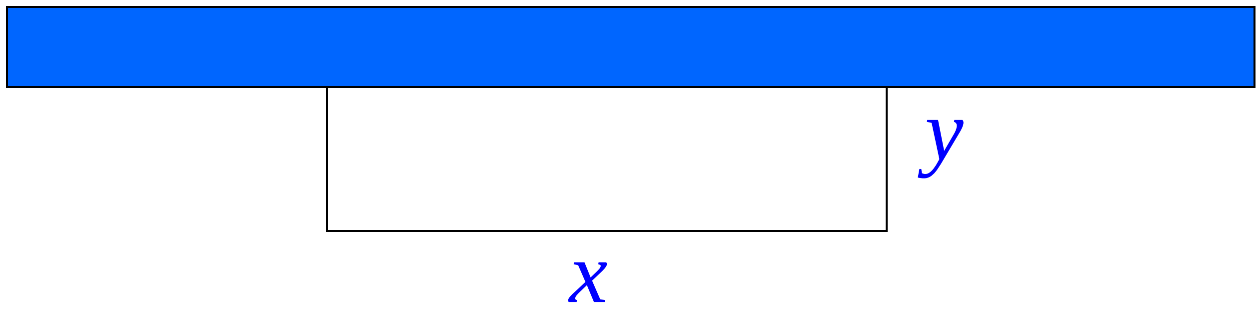
Draw a picture and make some variable assignments.



Find area as a function of a single variable.



$$3600 = x + 2y \Rightarrow x = 3600 - 2y$$

$$A = xy = (3600 - 2y)y = 3600y - 2y^2, \quad 0 \le y \le 1800$$

$$A'(y) = 3600 - 4y$$

$$A'(y) = 0 \Rightarrow y = 900$$

$$A''(900) = -4 \Rightarrow \text{ local maximum}$$

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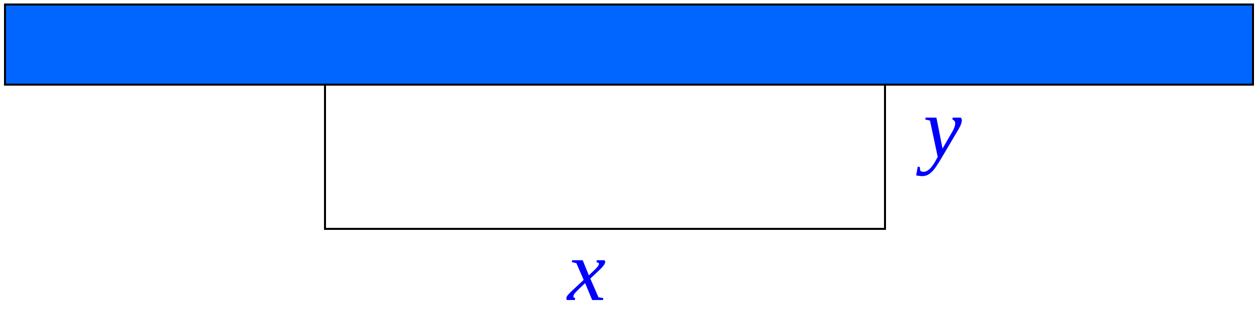

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$$A'(y) = 3600 - 4y$$

$$A'(y) = 0 \Rightarrow y = 900$$

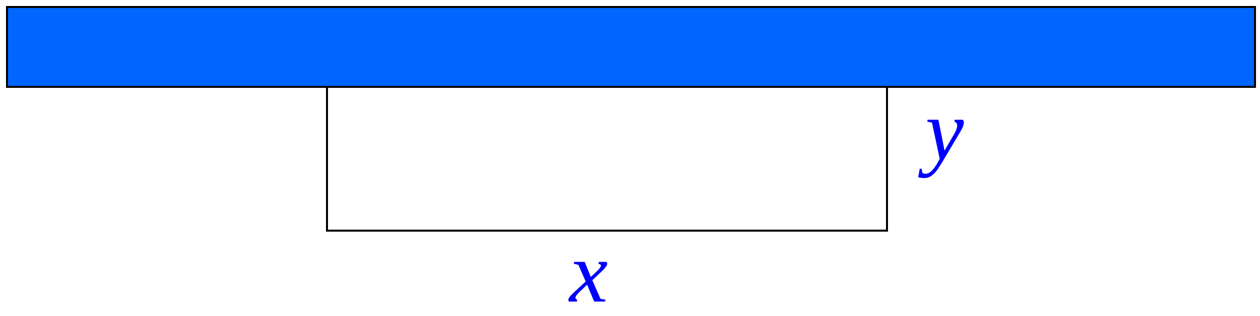
$$A''(900) = -4 \Rightarrow \text{ local maximum}$$

$$A(0) = 0$$

$$A(1800) = 0$$

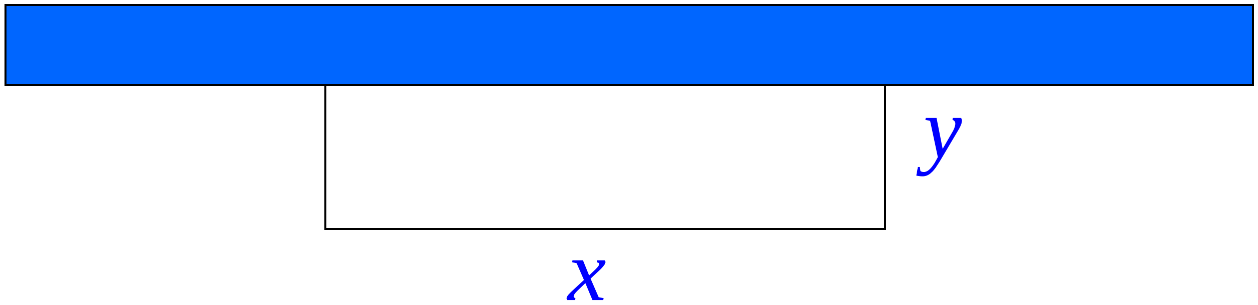
$$A(900) = 1,620,000\ ft^2$$

Find area as a function of a single variable.



The area of the rectangular pen is maximized at $1,620,000\ ft^2$ when the length is $1800\ ft$ and the width is $900\ ft$.

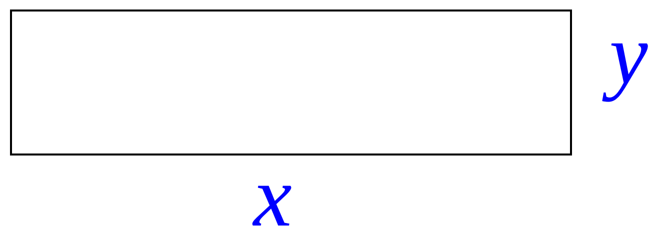
Find area as a function of a single variable.



The area of the rectangular pen is maximized at $1,620,000\ ft^2$ when the length is $1800\ ft$ and the width is $900\ ft$.

Can you make a conjecture about what the dimensions should be if you have *N* feet of fencing?

Find area as a function of a single variable.



What dimensions would maximize the rectangular area if you had to fence in all four sides?